

Review Paper

Comments on the interpretation of deformation textures in rocks

H.-R. WENK

Department of Geology and Geophysics, University of California, Berkeley, CA 94720, U.S.A.

and

J. M. CHRISTIE

Department of Earth and Space Sciences, University of California, Los Angeles, CA 90024, U.S.A.

(Received 23 August 1990; accepted in revised form 26 April 1991)

Abstract—In rocks that undergo ductile deformation, preferred orientation develops as a result of intracrystalline slip and mechanical twinning. The orientation distribution is a consequence of the microscopic mechanisms and of the strain path. It can be used to get some insight into the deformation history; however it is never unique. The interpretation relies largely on polycrystal plasticity theory. The concepts of stress equilibrium and strain compatibility, which are two extreme assumptions made to model deformation, are discussed. New approaches such as the viscoplastic self-consistent theory are a compromise and may be applicable to mineral systems which display a high degree of plastic anisotropy. Important extensions allow for heterogeneous deformation in the polycrystal from grain to grain and even within grains in correspondence with microstructural observations. All these theories defy the popular notion which is becoming entrenched in the geological literature, that the microscopic slip plane normal aligns with the axis of maximum principal compressive stress, and that in simple shear the crystallographic slip plane rotates into the macroscopic shear plane and the slip direction into the macroscopic shear direction, an orientation referred to by geologists as 'easy glide'. It is emphasized that future work on texture development of rocks should be based on rigorous physics rather than ingenious intuition, in accordance with an old recommendation of Walter Schmidt.

INTRODUCTION

THE importance of rock fabric as an indicator of geologic history was recognized by early petrologists. D'Hallo (1833, p. 227) and Cotta (1946, p. 117) used the terms *structure* and *texture* to describe properties of polycrystalline materials, but Naumann (1850, p. 443) was the first to introduce clear definitions that distinguished structural properties such as grain size and grain shape and textural properties which relate the directional arrangement of crystals. In this paper the term 'texture' is used interchangeably with 'crystallographic preferred orientation' which is different from the more general usage in petrography where 'texture' implies also grain size, grain shape and interrelations, and crystallinity. This should cause no ambiguity. The various aspects of 'texture' are intimately related to deformation, which is one of the most important processes in the Earth. It was the Austrian and German school of mineralogists and petrologists that initiated the quantitative study of fabric development. Some landmarks include the discovery of mechanical twinning in calcite by Mügge (1883) and slip in various minerals (e.g. Mügge 1898, Johnsen 1918, Veit 1922). In fact, Mügge's (1898) paper is still the only source reference for deformation mechanisms of many minerals. Such contributions to the study of deformation mechanisms were surprisingly quantitative and have remained relevant to this day.

Investigations on single crystals were extended to

deformed polycrystals, and fabric analysis blossomed under the direction of Bruno Sander in Innsbruck and Walter Schmidt in Berlin (e.g. Sander 1911, 1923, 1930, Schmidt 1912, 1932). These two scientists made the first serious attempts to interpret the development of crystallographic preferred orientation in deformed rocks, but each approached the problem with an entirely different philosophy. This point is important because 60 years later it remains at the root of the modern controversy about texture interpretation. Sander (e.g. 1931, p. 139: "man müsste dem induktiven Weg der Gefügeanalyse von Gesteinen gegenüber der technologischen Deduktion noch immer das erste Wort lassen . . .") maintained that an empirical-comparative approach was necessary because of the enormous complexities which preclude an analytical solution. Schmidt, on the other hand (e.g. 1927a, 1931), insisted that any interpretation needs to be based on physical principles. At the time, polycrystal plasticity theory was in its infancy: it would take a generation to develop, and only with the recent advent of high-speed computers can the deformation of polycrystals be modelled more or less quantitatively. Ironically, it was Schmidt's (1927b, p. 335: "Als Leitfaden der Erklärung muss die Erfahrung der Metallographen dienen, dass die Regelung in einer Einordnung der dichtest gesetzten Raumgittergeraden und Ebenen in die Richtungen der Umformung besteht, als welche bei uns Schieferungsebene und Richtung der Durchbewegung wirken.") oversimplified application of the physical con-

cept that a densely packed lattice direction orients itself into the macroscopic shear direction which introduced much of the confusion which still persists. Clearly, deformation is governed by physical processes, and in this respect Schmidt's approach should ultimately prevail, but only after painstaking quantitative theoretical and experimental work by materials scientists and physicists for which Reid (1973) provides a good review.

It is surprising how long it has taken geologists to accept these contributions. Even in the famous 1950 edition of Sander's *Einführung in die Gefügekunde der Geologischen Körper* there is no mention of E. Schmid (1928), who described single crystal deformation by introducing the concept of critical resolved shear stress, of Sachs (1928) who developed the first comprehensive polycrystal plasticity theory, or of Taylor (1938) who proposed a new plasticity theory which has since proved to be one of the most successful approaches. Nor did Sander mention dislocations, the motor of deformation and recrystallization, which had been proposed independently by Orowan (1934), Polanyi (1934) and Taylor (1934). Griggs and Turner in their Yule marble studies deserve credit for making geologists aware of progress in materials science (e.g. Turner *et al.* 1956). More recently Lister *et al.* (1978) applied the Taylor theory in the first quantitative model of fabric development in quartz.

Although texture analysis has gained renewed popularity among geologists, it has in some ways regressed, following a course depending either on plain intuition or simple geometrical constructions rather than well-founded physical concepts. The reasons for this regression lie with the physical complexity of the subject, which is now commonly beyond the background of a structural geologist. The most efficient approach is to rely on interdisciplinary collaboration with materials scientists. Most minerals are admittedly more complex materials than face-centered-cubic (fcc) metals, and classic theories cannot be applied directly. Rather than reject polycrystal plasticity theory for minerals having less than five slip systems and return to a phenomenological approach (as recommended, for example by Mainprice & Nicolas 1989, p. 176), we should perhaps strive to develop modifications to include more general cases.

The purpose of this paper is to clarify some misconceptions about interpretation of preferred orientations in rocks by explaining some concepts of polycrystal plasticity theory in non-mathematical language. Specifically two assumptions which are commonly made by geologists are challenged.

(1) In axial compression the slip plane normal orients itself parallel to the compression direction.

(2) In simple shear deformation, the 'easiest' slip direction orients itself parallel to the macroscopic shear direction and the 'easiest' slip plane orients itself parallel to the macroscopic shear plane, an orientation which geologists have called 'easy slip'. (The expression 'easy slip' or 'easy glide' was originally used by metallurgists to describe single slip deformation of copper single crystals with low work hardening, not an orientation with a high

'Schmid factor' or a low 'Taylor factor', discussed below.)

There are many examples in the literature where interpretations of preferred orientations in rocks are based on these assumptions, beginning with Schmidt (1927b), reiterated by Turner (1948, p. 246), and applied, among others, by Trommsdorff & E. Wenk (1963), H.-R. Wenk (1965), Etchecopar (1977), Schmid & Casey (1986), Etchecopar & Vasseur (1987), Mainprice & Nicolas (1989), Law (1990) and Law *et al.* (1990). The two assumptions mentioned above are not always incorrect, they are just not general. If, for instance, a crystal were in one of these preferred orientations, if it were deformed only on a single slip system, and if it were not constrained by neighbors, then it would—during deformation in compression and simple shear, respectively—indeed stay in the predicted preferred orientation. Such 'stable' end orientations appeal to the intuition because they are easy to visualize as resulting from the flow of rigid particles in a viscous medium: the slip directions are rods and slip planes platelets. However there is no obvious reason why crystals should rotate into these orientations, and not into others. Furthermore, most minerals have more than one slip system, i.e. the slip plane and the slip direction are not unique.

A real polycrystal which deforms and remains coherent has to fulfill both stress equilibrium and strain compatibility, and from the mechanical point of view there is a unique solution. However to solve this problem analytically is very difficult. Two extreme model approaches are most tractable. One assumes that each crystal deforms as an isolated grain with no knowledge about its neighbors, in a uniform stress field. This approach, which provides stress equilibrium, is a lower bound limit for stress. The second approach is to assume that the grain is totally controlled by its neighbors and suffers the identical strain of the aggregate, which guarantees compatibility at grain boundaries and is an upper bound limit for stress (Kocks 1970). These two concepts are illustrated with some mineral examples, mainly from our own work. Neither approach, it will be seen, predicts textures that conform with intuition. The response of real materials lies somewhere between the upper and lower bound solution, in a compromise to balance stress equilibrium and strain compatibility. As long as the two solutions are close—as is often the case in fcc metals—details are of little concern. However for minerals with low crystal symmetry and few slip systems, a compatibility and an equilibrium model can give very different answers and such systems need to be approached with more sophisticated theories that have recently been developed.

STRESS EQUILIBRIUM

In 1928, Schmid demonstrated experimentally for single crystals of Zn that in compression only that component of an applied force is active which is resolved

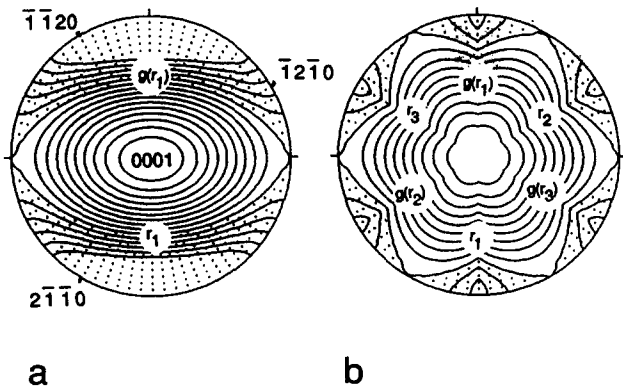


Fig. 1. Schmid factor $\cos \phi \cos \lambda$ for axial extension or compression deformation of calcite by r -slip in the negative sense, represented as contours of the unique stress axis in crystal co-ordinates. The $c = [0001]$ axis is in the center. (a) Single slip system with a slip plane normal $r_1 = (10\bar{1}4)$ and a slip direction $g(r_1) = [20\bar{2}1]$. (b) Combination of the three symmetrically equivalent systems r_1, r_2 and $r_3 \{10\bar{1}4\}\langle 20\bar{2}1 \rangle$. Maximum 0.5, minimum -0.5 , contour interval 0.05. The region with a negative Schmid factor in which no deformation can occur in compression in the negative sense $[20\bar{2}1]$ and in extension in the positive sense $[20\bar{2}1]$ is dotted. Equal-area projection.

on the slip plane and in the slip direction, leading to the $\cos \phi \cos \lambda$ law, or Schmid factor, where ϕ is the angle between slip plane normal and the applied compressional or tensional force and λ the angle between the slip direction and the applied force. The reason for this behavior is that slip occurs by movements of dislocations whose geometry is crystallographically controlled. According to this concept, if a stress is applied, no plastic deformation occurs until the shear stress resolved on the slip system reaches a critical limit, the critical resolved shear stress, which is a material constant. The crystal orientation is crucial for determining the necessary applied stress. Figure 1(a) shows the contoured Schmid factor (or 'resolved shear stress coefficient') for the compression direction, for slip on the slip plane $r_1 = (10\bar{1}4)$ in the slip direction $g(r_1) = [20\bar{2}1]$ (negative sense) in calcite. The z -axis (0001) is in the center, since r_1 and $g(r_1)$ are roughly at 45° to (0001), the largest Schmid factor (0.5) is at (0001) and this is the optimal orientation for deformation in axial compression. The Schmid factor is zero at 90° to r_1 and $g(r_1)$. The crystal cannot be deformed on this slip system outside this limit (dotted area). The situation is complicated by the fact that in all crystals except those of triclinic symmetry

there are several symmetrically equivalent systems, such as $r_1 = (10\bar{1}4), r_2 = (\bar{1}104)$ and $r_3 = (0\bar{1}14)$ in calcite. In Fig. 1(b) the Schmid factors for the three slip systems are combined, selecting for contouring in the superposition always the highest value corresponding to the active slip system. There may also be slip on symmetrically non-equivalent systems such as $r = \{10\bar{1}4\}\langle 20\bar{2}1 \rangle$ and $f\{01\bar{1}2\}\langle 20\bar{2}1 \rangle$ in calcite. This is because activation depends not only on the Schmid factor but also on the relative value of the critical resolved shear stress for each system.

Another consideration is the sense of slip. During slip, local bonds are broken and exchanged. Even though many minerals are centrosymmetric, the deformation across the slip plane is not (Fig. 2), and the critical shear stress for one sense (Fig. 2a) may be different from that for the other sense (Fig. 2b). For most materials this difference is not very well established, mainly through lack of experimental data rather than absence of the effect. One of the best examples for a difference in sense is calcite, where slip on $\{10\bar{1}4\}$ in the negative sense $\langle 20\bar{2}1 \rangle$ is preferred over that in the positive sense $\langle 20\bar{2}1 \rangle$ (Turner *et al.* 1954) (Fig. 2c). Obviously mechanical twinning can only occur in one sense (Fig. 3).

During slip an unconstrained crystal changes its shape but does not rotate (Fig. 4b). However, in a compression experiment the ends of the crystal must remain in contact with the piston. Accordingly it rotates the slip plane normal toward the compression direction (Fig. 4c). Similarly in tension rotation must accompany deformation if the pistons remain coaxial. In polycrystals similar constraints, imposed by neighbors, are responsible for rotations which lead to the development of preferred orientations. Let us turn to calcite again and assume that it deforms only by slip on $r^- = \{10\bar{1}4\}\langle 20\bar{2}1 \rangle$. In Fig. 5 rotations of the compression direction are represented in inverse pole figures which represent a specimen axis with respect to crystal co-ordinates rather than a pole figure which represents a crystal axis with respect to specimen co-ordinates. Inverse pole figures are particularly useful for axial deformation where only a single specimen axis (compression or extension) is relevant.

Each grain in a polycrystal deforms according to the external stress. As we increase the applied stress the

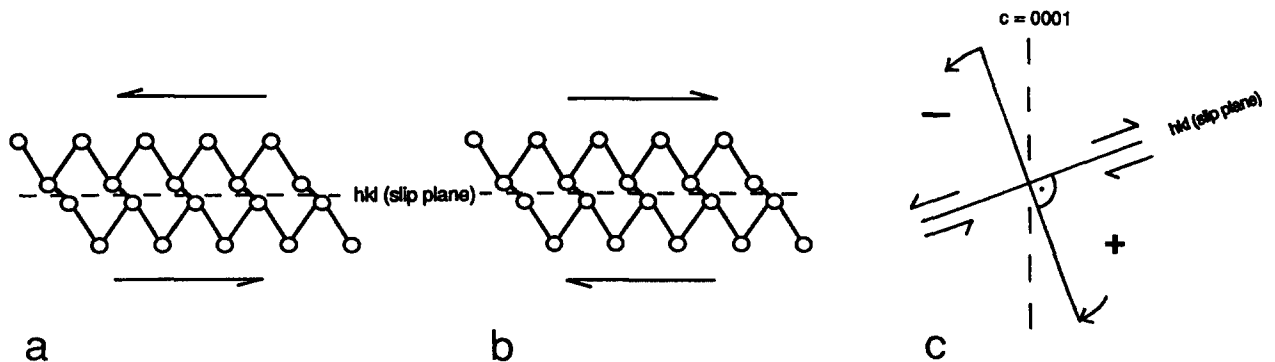


Fig. 2. (a) & (b) Schematic sketch of a crystal structure across a slip plane to illustrate the lack of centrosymmetry which explains the difference in strength for positive and negative sense of shear. (c) Definition of the positive (+) and negative (-) sense of slip relative to the $c = [0001]$ axis in calcite (after Turner *et al.* 1954).

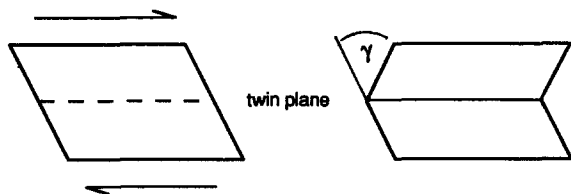


Fig. 3. Mechanical twinning can occur only in one sense as prescribed by the crystal structure. The twinning shear γ is indicated.

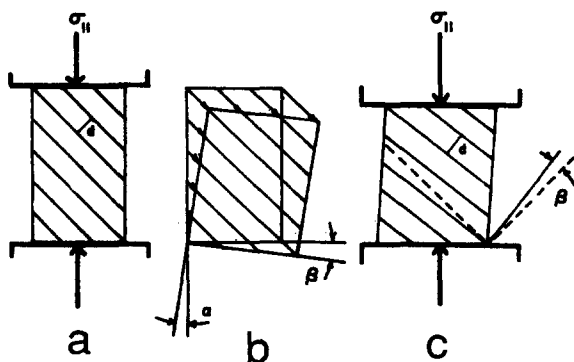


Fig. 4. In a compression experiment a rectangular crystal (a) deforms by slip into a parallelogram (b). Since pistons keep two sides of the crystal parallel, a lattice rotation β is imposed relative to external stress co-ordinates σ_{11} .

grains strain only elastically. Then, at some point those grains with an optimal Schmid factor for slip on r_2 in the outlined sector start to deform with rotations of the compression direction towards $r_2 = (\bar{1}104)$. As they rotate they become less favorably oriented; for deformation to proceed, the stress must be increased. As we increase stress, the resolved shear stress increases on other slip systems and grains with different orientations also become active, all rotating along trajectories indicated in Fig. 5(a). At some point compression axes reach

the border between $r_1 = (10\bar{1}4)$ and $r_2 = (\bar{1}104)$ slip. There is probably some overshoot, with r_2 dislocations still active in the r_1 field, but then r_1 dislocations become active and the combined rotation trajectory of symmetrical r_1 and r_2 slip is the vector sum (Calnan & Clews 1950, Turner *et al.* 1956, Wenk *et al.* 1973). Rotations follow the e - m borderline and reach a stable orientation at its intersection with the great circle connecting the two active slip plane normals. This is in the vicinity of $e_3 = (0\bar{1}18)$, which, according to this model, constitutes a 'stable orientation' (assuming that only r -slip is active). Note that neither the active slip plane r_1 nor the slip plane r_2 are normal to the compression axis. The compression axis can never reach the slip plane normal because at that point the Schmid factor is zero and an infinitely high stress would be required to activate slip. The Schmid model allowing for symmetrical duplex slip (which is present in most crystals except those that are triclinic and if the slip vectors and slip plane normals are parallel to unique symmetry axes such as $(010)[100]$ slip in olivine), thus predicts textures which are at variance with our 'natural intuition' and assumption 1 introduced above. In the case of calcite, the texture with a compression maximum at e is in reasonably good agreement with axial compression experiments at high temperatures on marble (Fig. 5b) (Spiers 1979) and limestone (Fig. 5c) (Wenk *et al.* 1973).

STRAIN COMPATIBILITY (TAYLOR THEORY)

The model described above for calcite is a rather crude approach. Only at very small deformations can a grain deform independently of its neighbors. When a dislocation reaches a grain boundary, it will project stresses into the neighboring grain, and the neighbor, depending on its orientation, will impose backstresses

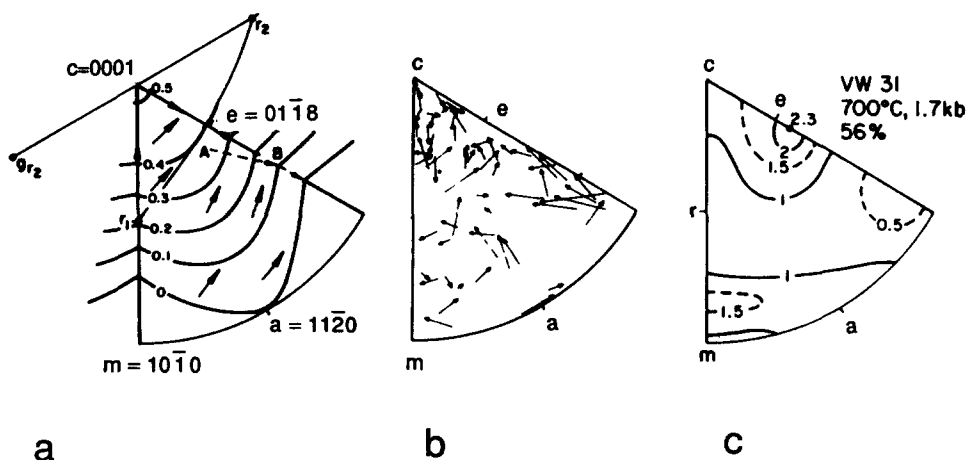


Fig. 5. (a) Rotations of crystals during axial compression of calcite which deforms by $\{10\bar{1}4\}\langle 20\bar{2}1\rangle$ slip. The Schmid factor for slip on r_2 is contoured in crystal co-ordinates (inverse pole figure) (cf. Fig. 1b). In axial compression the compression axis moves towards the pole of the slip plane (r_2 for the sector illustrated). Some rotations are indicated by arrows. Whenever the line c - m is crossed, r_1 slip becomes active and rotations are reversed towards r_1 . Compression axes along c - m undergo duplex slip on r_1 and r_2 and rotate towards the intersection with the great circle connecting r_1 and r_2 . This position is a stable orientation for r -slip according to the Schmid model. (b) Measured rotation trajectories of calcite crystals in a marble deformed in axial compression (after Spiers 1979). (c) Experimentally deformed, non-recrystallized limestone displays an orientation distribution which is in good correspondence with the model described in (a) (after Wenk *et al.* 1973).

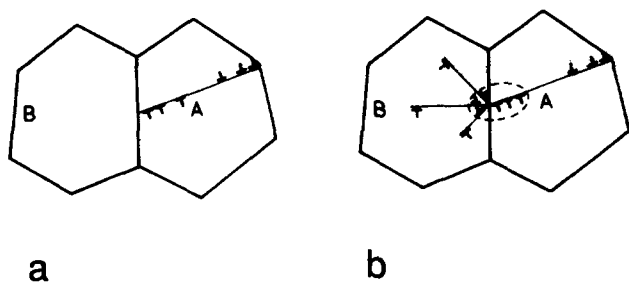


Fig. 6. Dislocations are active in grain A and move towards the grain boundary with B (a). As they reach the boundary they create stresses and back stresses which activate new dislocations both in B and A (after Leffers 1981).

(Fig. 6). If neighbors have a similar orientation, back-stresses may be assumed to be small; if the orientation is different, and the neighbor cannot deform, these stresses may be very high. To solve this problem satisfactorily, one needs to know the orientation of each grain and the orientation and morphology of all neighbors. Then one could approach the problem with the finite element theory, but even for two-dimensional models the computer requirements are prohibitive (e.g. Asaro & Needleman 1985, McHugh *et al.* 1989). Furthermore, crystallographic slip and texture development are three-dimensional processes and any two-dimensional model is not very realistic. Also, in most cases we do not have an adequate description of the initial fabric. Only orientations of grains are measured and the orientation, size and shape of neighbors are unknown. It is not inconceivable that Sander's "Achsenverteilungs Analyse (AVA)" (axis distribution analysis) (Ramsauer 1941, Sander 1950) which deals with orientation correlations and local environments will be revived for future work and become accessible to quantitative interpretation (e.g. Adams 1986), particularly with the recent possibility of measuring orientation distributions with the scanning electron microscope (e.g. Lloyd *et al.* 1987).

In 1938 Taylor proposed an ingenious and simple solution to avoid gaps and overlaps at grain boundaries: to impose homogeneous deformation; i.e. all grains should undergo the same shape change regardless of

their orientation. Figure 7 illustrates a section through a polyhedral microstructure consisting of two phases. All grains have been deformed homogeneously in pure shear to an equivalent strain of 50%. During such deformation grain boundaries and vertices remain intact. Homogeneous deformation requires that each grain be able to undergo the macroscopically imposed shape change. Since the 3×3 strain tensor is symmetrical, and because of volume preservation the product of the diagonal elements remains constant, there are five variables which require five degrees of freedom. Each independent slip system contributes one degree of freedom and up to five independent slip systems are necessary to produce an arbitrary shape change (e.g. Groves & Kelly 1963). For special deformation conditions and crystal orientations, fewer systems may suffice (e.g. two in pure shear, in axial compression at least three are necessary), but never more than five need to be activated. In many crystals there are more than five potential slip systems. For example, in fcc metals $\{111\}\langle 110 \rangle$ slip involves 48, and in calcite r and f slip and e twinning add up to 21. A major problem lies in selecting the five active systems in each grain. Taylor proposed selecting that combination of five which requires the least amount of plastic work; this depends on critical resolved shear stresses and orientation. Quite analogous to the Schmid factor for single slip, a Taylor factor can be defined (e.g. Kocks 1970, Gil Sevillano *et al.* 1980, Van Houtte & Wagner 1985). Whereas the Schmid factor is large if a slip system is favorably oriented for slip, the Taylor factor is large for orientations which are unfavorable. In early applications of the Taylor theory an artificial ambiguity was introduced by assuming strict strain rate insensitivity of the slip systems. In fcc metals with $\{111\}\langle \bar{1}10 \rangle$ slip, for example, eight systems instead of five are equally likely to be activated. This problem vanished when Hutchinson (1970) and Asaro & Needleman (1985) introduced rate sensitive constitutive equations which can be incorporated in a modified Bishop-Hill (1951) approach (e.g. Kocks & Canova 1981).

In the equilibrium model a grain can deform as soon as

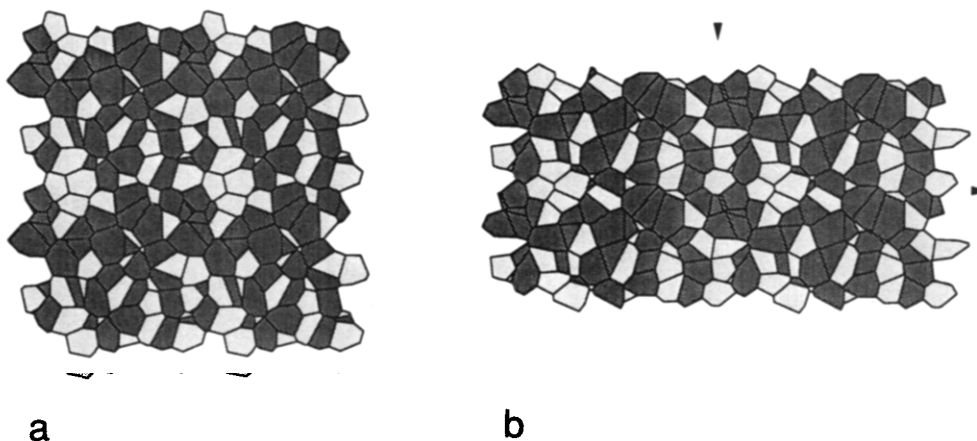


Fig. 7. Two-dimensional section through a three-dimensional microstructure consisting of two phases. Polyhedra were produced by generating 200 grain centers at random and constructing bisecting planes between pairs of points. The grain polyhedron is defined as the inner envelope of these planes around each center. The microstructure is repeated by translation. (a) Microstructure before deformation. (b) Microstructure after 50% homogeneous deformation in pure shear.

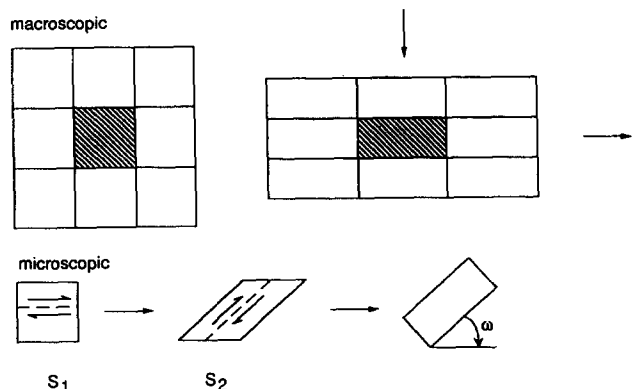


Fig. 8. Two-dimensional homogeneous deformation of a microstructure with originally square shaped grains. The macroscopic strain supplies the same rectangular space for each grain. In two dimensions two slip systems, s_1 and s_2 , are sufficient to deform each grain to this rectangular shape; however, depending on crystal and slip system orientation, these rectangles have a different orientation and need to be rotated to fit into the provided space. The rotations ω produce preferred orientation.

the critical shear stress on one slip system is reached. For the Taylor compatibility theory the stress needs to be increased until the critical shear stress on all five systems is reached. Therefore the yield stress for compatibility is higher ('upper bounds') than for equilibrium ('lower bounds') as can be illustrated with the single crystal yield surface. In an equilibrium model the weakest systems control deformation, whereas in the Taylor model the stronger systems contribute equally and determine the yield stress.

Once the five active slip systems have been selected, shears on each system can be determined and, correspondingly, the rotation of each grain. In the case of the Schmid model, at least for compression geometry, rotations can be relatively easily visualized. For Taylor conditions our brains are unable to predict the five active systems, determine the shear on each and derive from them the effective grain rotation. This can only be done with the help of high-speed computers, which is the main reason why the Taylor theory was never quantitatively applied for 30 years after it was first proposed. Only a few years ago large mainframe computers were still a prerequisite, but today most of the calculations can be performed efficiently on affordable personal computers.

Within the Taylor model we can visualize rotations as follows: according to the compatibility criterion, the macroscopic strain prescribes for each grain a polyhedral shape in the polycrystal (Fig. 8, top). On the five slip systems we deform a crystal to this shape; however, in general the resulting polyhedron needs to be rotated to fit into the prescribed cavity (Fig. 8, bottom). This rotation of a grain leads to development of preferred orientation. Rotations do not, in general, go towards an 'easy slip' orientation, as is intuitively clear for compression and extension.

The Taylor theory is illustrated with simple shear deformation of a quartz aggregate assuming basal $(0001)\langle\bar{1}2\bar{1}0\rangle$ (normalized critical shear stress $\tau_c = 1$), prismatic $\{10\bar{1}0\}\langle\bar{1}2\bar{1}0\rangle$ ($\tau_c = 0.4$), and pyramidal $\{10\bar{1}1\}\langle\bar{1}2\bar{1}0\rangle$ ($\tau_c = 3$) and $\{10\bar{1}1\}\langle 2\bar{1}\bar{1}3\rangle$ ($\tau_c = 6$) slip. This corresponds to deformation at moderate to high

temperature (model β of Wenk *et al.* 1989a), where prismatic slip is preferred over basal slip. Several features are observed in (0001) and $(1\bar{1}20)$ pole figures after a shear strain of $\gamma = 1.732$ (Fig. 9, left side). The rotation trajectories illustrate that crystals rotate relative to the shear plane. Also they document the lack of stable orientations in simple shear in accordance with recent theoretical investigations (e.g. Canova *et al.* 1984, 1988, Toth *et al.* 1988). Crystals constantly rotate, particularly fast for rate-sensitive materials with a small stress exponent (e.g. 3 in the case of quartz, Heard & Carter 1968), which is contrary to the intuition that the slip plane orients itself into the shear plane (e.g. Etchecopar 1977, Schmid & Casey 1986, Law *et al.* 1990). The reason for texture development in simple shear is that rotations occur at different rates for different orientations, with maxima in the pole figures corresponding to orientation regions in which rotations are slowest. The orientation distribution in simple shear is in a dynamic condition. Notice particularly that in these simulated quartz textures there is neither an alignment of the dominant slip planes $\{10\bar{1}0\}$ and (0001) in the macroscopic shear plane nor an alignment of the dominant slip direction $\langle\bar{1}2\bar{1}0\rangle$ in the macroscopic shear direction, as suggested by assumption 2. Instead, the distribution is asymmetric.

This is confirmed by experiments for quartz at low strain (e.g. Dell'Angelo & Tullis 1989) and for flint (Mainprice & Paterson in press). It is also true for such materials as fcc metals, where the slip plane $\{111\}$ is inclined to the shear plane (Fig. 10a). In halite where $\{110\}$ is the favored slip system, there is a $\{110\}$ maximum (Fig. 10b) and in calcite where $\{10\bar{1}4\}$ is the dominant slip system, $\{10\bar{1}4\}$ poles display a maximum normal to the shear plane (Fig. 10c). But in both cases the slip direction is inclined to the shear direction. None of these materials develop orientation distributions consistent with the 'easy glide' hypothesis.

Maxima in the orientation distribution are dependent on slip systems and deformation history, and may represent a *dynamic state* or a *stable state*. In the former case textures never become exceedingly strong; in the latter they reach stronger concentrations with increasing deformation. Because of the complexities outlined above, both the Taylor and the Schmid models predict that there is no simple relationship between orientation of maxima in the texture and their disposition relative to macroscopic strain co-ordinates and the active slip systems. Because this issue has been at the root of much confusion, it is useful to discuss another concept, *ideal orientations*.

IDEAL ORIENTATIONS AND THE ORIENTATION DISTRIBUTION FUNCTION

Increasingly, three-dimensional representations are being used to express the predicted or measured relationship between the reference sample co-ordinate system and the orientation of crystallites. Most com-

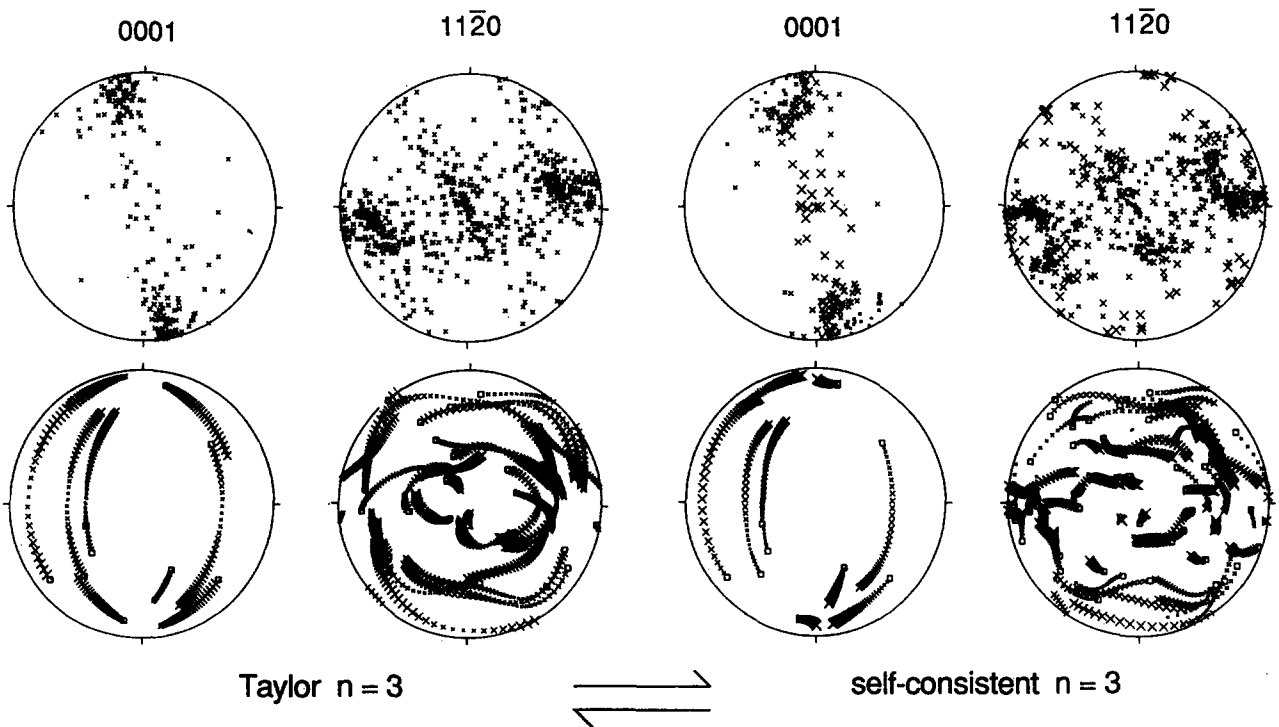


Fig. 9. Simulated (0001) and (1120) pole figures for quartz deformed at moderate to high temperature conditions (prescribed by the critical resolved shear stresses for the slip systems, model β of Wenk *et al.* 1989a) in simple shear, and assuming a viscoplastic behavior described by a power law with a stress exponent $n = 3$. Taylor theory (left) and viscoplastic self-consistent theory (right). Top: 200 grains after a shear of $\gamma = 1.732$. The size of symbols is proportional to the accumulated change in grain shape (corresponding to plastic work). Bottom: rotation trajectories of 10 grains with 5% equivalent strain increments. Square represents the initial orientation. The size of symbols is proportional to the Taylor factor (effective stress). The sense of shear is indicated at the bottom. Equal-area projection.

monly the probability function of the three Euler angles (defining with three rotations the relationship between crystal and specimen reference axes; Bunge 1965, Roe 1965) is being employed. This is known as the *orientation distribution function* or ODF, and can be—within limits—calculated from measured pole figures (Bunge 1969). Such representations have been called quantitative (e.g. Bunge 1969) or complete (e.g. Schmid & Casey 1986) descriptions of textures. Both terms are misleading.

A function cannot be more quantitative or more

complete than the data it is calculated from. ODFs are generally derived from pole figures. It is correct that sometimes a single pole figure is not sufficient to define the orientation relationship. For example, a (0001) pole figure of quartz does not give information about the orientation of *a*-axes. However, a (10 $\bar{1}$ 4) pole figure of calcite defines the orientation relation completely and is sufficient to derive from it an ODF. There are several deficiencies of ODFs. Ambiguities arise during the pole figure inversion due to the impossibility of assigning directionality to poles (e.g. Wenk *et al.* 1988). Also,

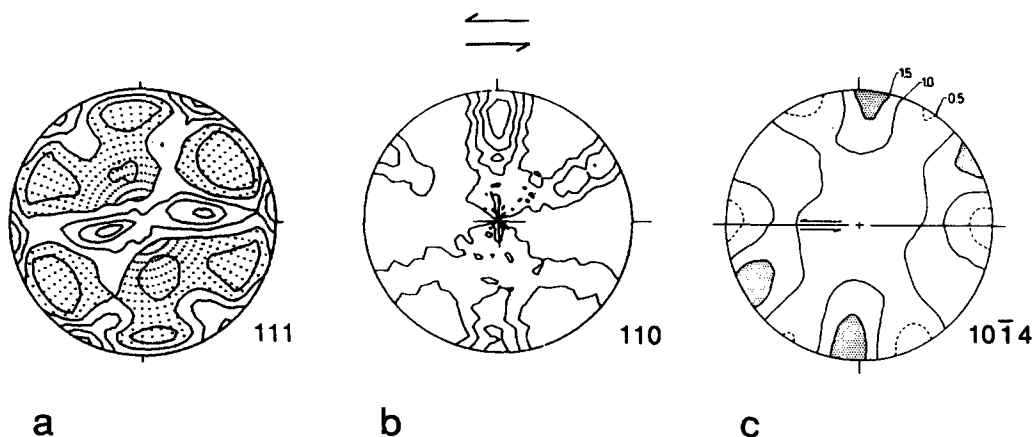


Fig. 10. Experimental simple shear pole figures for (a) (111) of Ni (fcc), minima are dotted (Hughes & Wenk 1988); (b) (110) halite (Franssen & Spiers 1990); and (c) (10 $\bar{1}$ 4) of calcite (maximum is shaded) (Kern & Wenk 1983). The shear plane is horizontal and the sense of shear is indicated. In none of these examples is there both an alignment of the crystallographic slip plane with the shear plane and the crystallographic slip direction with the shear direction as is predicted with the 'easy glide' hypothesis.

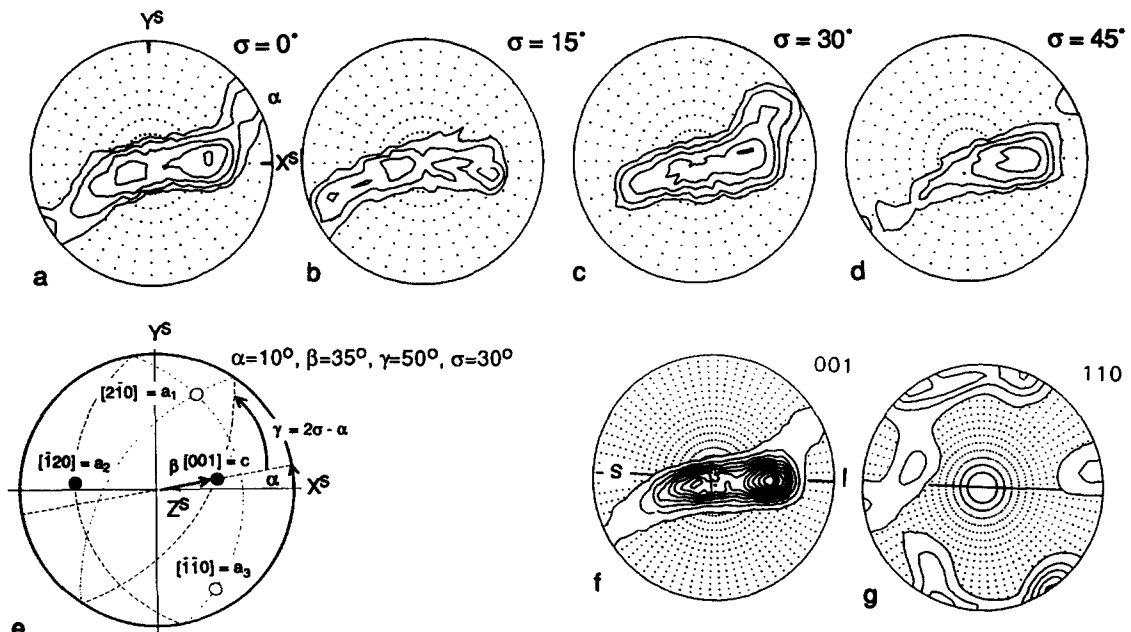


Fig. 11. Top (a)–(d): orientation distribution of quartz in a recrystallized mylonite from the northern Bergell Alps represented in spherical co-ordinates as $\sigma = (\alpha + \gamma)/2$ sections which show a minimum of distortion. X^S is chosen to be parallel to the lineation and Y^S parallel to the pole of the schistosity. Since there is little change in the pattern with σ , a -axes are only moderately aligned. Contour levels are 1, 2, 4, 8, 16 multiples of a random distribution, dotted $10^\circ \times 10^\circ$ grid below 1 m.r.d. Bottom (e): interpretation of a particular orientation maximum ($\alpha = 10^\circ$, $\beta = 35^\circ$, $\sigma = 30^\circ$, $\gamma = 2\sigma - \alpha = 60^\circ$) in terms of crystal orientation relative to specimen axes X^S , Y^S , Z^S . Roe–Matthies convention for Euler angles α , β and γ . Except for dotted lines and open circles in (e) all features are equal-area projections of upper hemisphere. (f) & (g) (0001) and (1120) pole figures for comparison, schistosity (s) and lineation (l) are indicated in (f). Contour interval in (f) and (g) is 1 m.r.d., dots are a $5^\circ \times 5^\circ$ grid below 1 m.r.d.

numbers are not really quantitative unless errors and uncertainties are assigned (Matthies *et al.* 1988). Many ODFs are incomplete because of the difficulty in resolving the true crystal symmetry. In the case of quartz, all published ODFs have textures of left-handed and of right-handed crystals superposed. Furthermore, ODFs only describe orientations of crystals and contain no information on orientation relationships between neighboring grains and are therefore ‘incomplete’ (e.g. Adams 1986). Despite these limitations, ODFs have distinct advantages compared to pole figures. For example the ODF is necessary as an analytical method for texture simulations and for calculations of physical properties.

As a method of representation, the ODF is not totally satisfactory. Not only have several specifications of Euler angles, signs of angles and Greek symbols been used (such as Bunge: ϕ_1, Φ, ϕ_2 ; Canova: ω, θ, ϕ ; Casey: Ψ_1, Φ, Ψ_2 ; Kocks: Ψ, Θ, ϕ ; Matthies: α, β, γ ; Roe: Ψ, Θ, Φ) which are by no means trivial to relate (e.g. Kocks 1988). The ODF is also difficult to read and visualize, particularly if the density distribution of angles is represented in traditional Cartesian co-ordinates. The more natural representation of angles in spherical co-ordinates (Wenk & Kocks 1987, Wenk *et al.* 1987a, p. 733) has been helpful and provides greater ease of visualization. An example is illustrated in Fig. 11. But even so, it is conceptually very difficult to transform an ODF which is given in a certain sample reference frame to one with different sample reference axes. This is particularly serious for geological samples where assignments of foliation, cleavage and schistosity are often ambiguous.

In the case of pole figures on stereographic or equal-area projections, such rotations of sample co-ordinates can be easily visualized on the projection sphere. Deviations from orthorhombic symmetry as an indicator of non-coaxial deformation, for example, can be measured on a pole figure with the help of an equal-area net. This is not possible with an ODF without complicated transformations. Whereas ODFs are indeed more compact than pole figures, fabric diagrams (e.g. of quartz c -axes or calcite twin planes) remain very useful and no structural geologist should feel inhibited to use them because he does not have access to the elaborate mathematical framework to calculate ODFs.

The three-dimensional pole density distribution often contains a small number of strong peaks representing the orientation of a concentration of crystals. In the case of the mylonitic quartz texture in Fig. 11 there is an orientation concentration with a broad tube shape and a maximum at $\alpha = 10^\circ$, $\beta = 35^\circ$ and $\sigma = (\alpha + \gamma)/2 = 30^\circ$ shown in spherical co-ordinates as σ -sections with minimum distortion (Helming *et al.* 1988) and using the convention of Matthies *et al.* (1988). Two Euler angles (α, β) describe the direction of the c -axes, and the third (γ) the direction of the a -axes. We can immediately read from the projections the orientation of crystals which contribute to that maximum. As the diagram illustrates, it is possible to visualize graphically the orientation of these crystals relative to the sample, which cannot be done from pole figures (e.g. Schmid *et al.* 1981).

Long before ODFs were invented, metallurgists characterized textures by ‘ideal orientations’ (e.g. Grewen & Wassermann 1955). Wassermann & Grewen

(1962) used Miller indices to classify peaks in texture patterns of rolled metals. They determined which lattice plane (hkl) lies preferentially in the rolling plane and which lattice direction $[uvw]$ lies in the rolling direction. This scheme of ideal orientations makes it easy to visualize the orientation of crystals contributing to a concentration in pole figures and ODFs, and has greatly gained in importance since three-dimensional orientation distributions became available and could be expressed by model functions such as combinations of Gaussians (Virnich *et al.* 1978). Some examples of 'ideal orientations' for fcc metals are shown in Table 1. Geologists could use a similar scheme of ideal orientations using mesoscopic fabric co-ordinates as a reference system. One difficulty is that foliation and lineation are less clearly related to strain co-ordinates than the normal and rolling directions in a metallurgical rolling process. In addition, orientation distributions for geological materials often cannot be deconvoluted into simple Gauss components but form complicated tubes of high orientation density in the ODF (e.g. Schmid & Casey 1986 for quartz, and Takeshita *et al.* 1987 for calcite).

The essential point is that ideal orientations are a *method of description*, and even though Miller indices are used, they have nothing to do with active slip systems. In rolled fcc metals there are about three 'ideal' orientations (copper, brass and Goss) depending on material and deformation conditions. The only active slip system, $\{111\}\langle 1\bar{1}0\rangle$, is *not* one of them (Table 1). Similarly in a rock there may be an orientation concentration with a lattice plane (hkl) in the foliation plane and a lattice direction $[uvw]$ in the lineation direction but this is no evidence whatsoever about active slip systems. In 1928 it was permissible for Walter Schmidt to propose on intuition that in quartz the slip plane $\{10\bar{1}1\}$ is in the foliation plane and the slip direction $\langle 11\bar{2}0\rangle$ is the lineation direction. Since then, theories and experiments have demonstrated that this is not the case, and it is unpardonable if this mistake is perpetuated. Interestingly, Schmidt (1927b, p. 335: "Hier muss eine Konkurrenz zweier verschiedener Gitterebenen mitspielen, die je nach dem Gang der Reibungsverhältnisse mit den äusseren Bedingungen zu verschiedenen Endlagen führen kann") had seen the possibility of multiple slip, but

neither he nor later interpreters of quartz fabrics elaborated on it.

EFFECTS OF HETEROGENEOUS DEFORMATION

Stress equilibrium at the expense of strain compatibility, and homogeneous deformation at the expense of equilibrium, are extreme assumptions which may be useful for model calculations only as long as the two solutions give similar results. Depending on the plastic properties of the material and on the deformation history, deformation of a polycrystal can be modelled better with a theory which is closer to equilibrium (lower bound) or closer to compatibility (upper bound). It is interesting that the equilibrium theory is based on deformation experiments on crystals of hexagonal zinc (Mark *et al.* 1923, Polanyi 1925, Weissenberg 1925, Schmid 1928), whereas Taylor was inspired by experiments on highly symmetric aluminum (Taylor & Elam 1923). The plastic anisotropy of polycrystals can be assessed by comparing Taylor factors for different grain orientations. The Taylor factor is proportional to the stress required to deform a crystal in an arbitrary orientation and can therefore never be lower than the critical resolved shear stress of the weakest slip system. In fcc metals the Taylor factor for the 'softest' orientation is about a factor of 2 lower than for the 'hardest' orientation. By contrast the range in Taylor factors for quartz is greater than an order of magnitude (Wenk *et al.* 1989a). In the latter case, 'soft' grains are very likely to deform more and at a faster rate than 'hard' grains, leading to heterogeneous deformation which is indeed observed in experimentally deformed quartz aggregates and in quartzites deformed at low to moderate metamorphic grade.

Heterogeneous deformation has been approached with different theories. Some are within the Taylor framework. The 'relaxed constraints model' (Honneff & Mecking 1978, Kocks & Canova 1981) allows for arbitrary deformation on small grain faces, such as occur on flattened or elongated grains. For a flat grain such as that in Fig. 12, misfits on the small faces B and C are negligible. Equations which prescribe the shears $d\varepsilon_{13}$ and $d\varepsilon_{23}$ can be neglected, requiring only three slip systems to be activated instead of five. This effectively favors soft slip systems with a low critical resolved shear stress over hard ones. The relaxed constraints model was successful in eliminating the artificial 'Taylor component' which persistently appeared in texture simulations of rolled fcc metals, but does not exist in reality. Obviously 'relaxed constraints' become meaningful only when crystals are already deformed or otherwise have an anisotropic grain shape. An aspect ratio of 2 is often chosen as a lower limit. A modification of 'relaxed constraints' has been applied successfully to simulations of high deformation symmetry such as curling in extruded body-centered-cubic wires (Hosford 1964) and axially compressed calcite (Wenk *et al.* 1986) where

Table 1. Some 'ideal orientations' in rolled cubic metals. $\{hkl\}$ is parallel to the normal direction (rolling plane) and $\langle uvw\rangle$ is parallel to the rolling direction

$\{hkl\}\langle uvw\rangle$	Name	Euler angles					
		Bunge convention			Roe-Matthies convention		
		ϕ_1 (°)	Φ (°)	ϕ_2 (°)	$\Psi = \alpha$ (°)	$\Theta = \beta$ (°)	$\Phi = \gamma$ (°)
$\{\bar{1}\bar{2}1\}\langle \bar{1}11\rangle$	copper	40	66	26	-50	66	116
$\{\bar{1}32\}\langle \bar{6}43\rangle$	S	28	58	18	-62	58	108
$\{011\}\langle \bar{2}11\rangle$	brass	35	45	0	-55	45	90
$\{011\}\langle 100\rangle$	Goss	0	45	0	-90	45	90
$\{025\}\langle \bar{1}00\rangle$	cube	0	22	0	-90	22	90

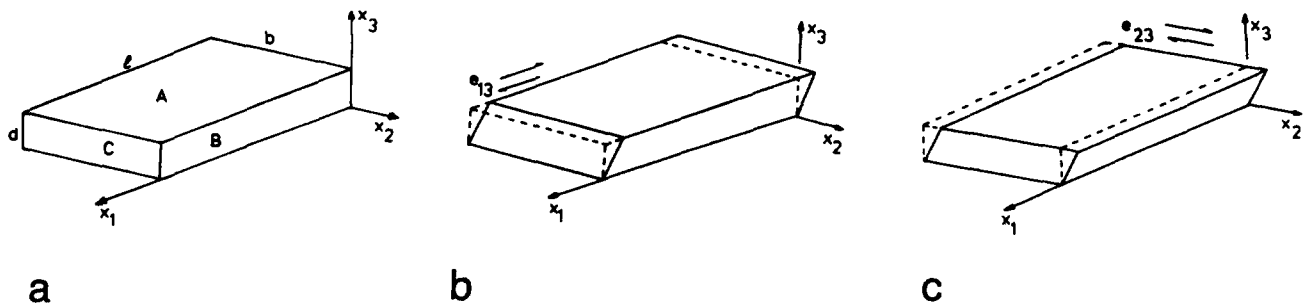


Fig. 12. In the full constraint Taylor theory compatibility needs to be attained across all boundaries A, B and C, which requires five slip systems. In the relaxed constraint model for flat grains, misfits across small faces B and C are allowed and equations which prescribe shears $d\epsilon_{13}$ (b) and $d\epsilon_{23}$ (c) are neglected. This reduces the number of required slip systems to three.

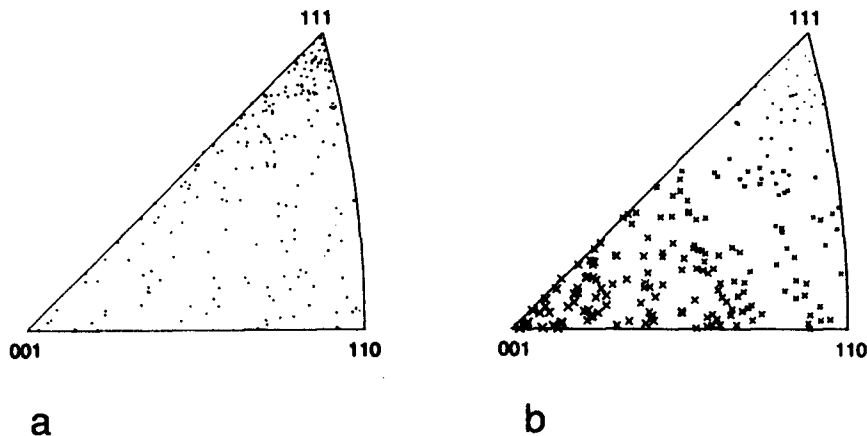


Fig. 13. Simulations of texture development in halite deformed in axial extension. Representation in inverse pole figures of the extension direction, 50% strain. The Taylor theory (a) predicts extension directions to move towards (111), whereas the viscoplastic self-consistent model (VPSC) (b) predicts rotations towards (001). Stereographic projection (Wenk *et al.* 1989b). Symbol sizes in the VPSC case indicate degree of deformation of individual grains.

observed textures could not be explained with the fully constrained Taylor theory.

Another extension of the Taylor theory is the 'cluster model' (Canova *et al.* 1985). This concept, from which results have not yet been published, allows a grain to deform heterogeneously, but homogeneity has to be maintained in an average cluster. A variation is the 'constrained hybrid' model of Parks & Ahzi (1990), in which each grain is surrounded by a thin amorphous film within which it has more freedom.

A different approach is the self-consistent theory, which maintains both equilibrium and compatibility, at least on the average over the whole polycrystal. The concept was originally developed for elasto-plastic deformations (Kröner 1961, Budiansky & Wu 1962, Hutchinson 1976). Recently a viscoplastic self-consistent theory (VPSC) was introduced by Molinari *et al.* (1987) to model large plastic strain. In the VPSC scheme, deviations in stress and strain rate of viscoplastically deforming grains from the macroscopically prescribed are minimized, taking account of interactions between neighbors. The theory can be viewed as a correction of the Taylor theory for self-consistency and neighbor interaction (Wenk *et al.* 1991). For plastically highly isotropic materials such as fcc metals, VPSC predictions agree closely with those of Taylor (Molinari *et al.* 1987), but if plastic anisotropy is significant, as in halite, results

may be quite different (Wenk *et al.* 1989b) (Fig. 13). For axial extension (purely constrictional deformation) the VPSC theory predicts a maximum at (100) in the inverse pole figure due to the activation of only the weak $(110)\langle\bar{1}01\rangle$ slip system, whereas Taylor predicts a maximum at (111) which is caused by $(111)\langle\bar{1}10\rangle$ slip required to maintain compatibility. In the self-consistent model, unfavorably oriented grains are allowed to deform more slowly, and each grain has the freedom—within self-consistent constraints—to follow an optimum strain path which may be different from the macroscopically imposed one. Indeed, in the case of extension of halite some grains barely deform at all, others undergo plane strain deformation, and only a small fraction deforms in pure extension geometry (i.e. constriction), as is illustrated with a Flinn diagram (Fig. 14). The VPSC deformation model has some very attractive features for structural geology. For example it is applicable to crystals with fewer than five slip systems, such as olivine (Takeshita *et al.* 1990) and polyphase materials such as peridotite (Wenk *et al.* 1991).

If grains deform differently, intergranular heterogeneity has to be compensated by intragranular heterogeneity for the polycrystal to remain coherent. Recently the VPSC scheme has been modified to allow for heterogeneous deformation within grains and was applied to a composite of quartz and relatively stiff platelets of mica

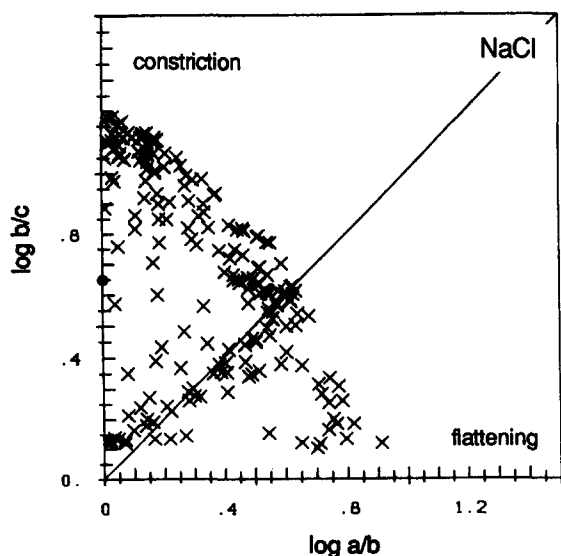


Fig. 14. Flinn diagram plotting predicted ratios of strain ellipsoid axis lengths for halite grains deformed in extension to 100% equivalent strain with the VPSC theory (cf. Fig. 13). Note the wide distribution of grain shapes. For Taylor, all grains plot at $\log a/b = 0$ and $\log b/c = 0.6$ (dot). The distance from the origin indicates the degree of deformation. Even though the macroscopic strain prescribes extension (dot) (i.e. constriction), many grains deform in plane strain (diagonal line) and some even in the field of flattening.

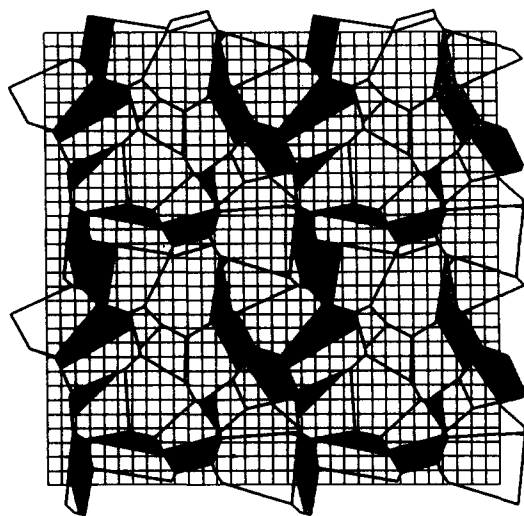


Fig. 15. Two-dimensional section through a random microstructure of a 75% quartz (light)-25% mica (dark) aggregate divided into cubic cells. Microstructure is repeated by translation (Canova *et al.* in press).

(Canova *et al.* in press). A microstructure of grains such as that in Fig. 7 is subdivided into cubic cells (Fig. 15). Each cell is specified by orientation, phase identity and stress state, and can deform differently, controlled by interaction with neighboring cells. Figure 16 illustrates (0001) pole figures for a quartz-mica composite deformed in axial compression to 50% strain. On the left are pole figures for pure quartzite and a 75% quartz-25% mica mixture predicted when the whole grain is forced to deform uniformly. Notice that the pure quartzite has a weaker texture than the mixture. Since in the model mica barely deforms (only basal slip is allowed,

preventing mica platelets from attaining an arbitrary shape change), all strain is accommodated in quartz, leading to larger rotations. On the right, each grain has been divided into domains which are allowed to deform differently. Each original grain orientation spreads with deformation into an increasingly wider distribution. Notice that here the pattern for pure quartz is less smooth than that for the mixture, due to interactions between 'strong' mica platelets and 'soft' quartz which forces quartz to deform heterogeneously. This behavior is more easily visualized by considering a single grain and the range of orientations that develop from it during deformation (Fig. 17). The spread is about twice as large for the quartz grain in the mixture as it is for the pure quartzite. Predictions with this domain scheme are in good accordance with observations that addition of stronger phases reduces the strength of preferred orientation of the more ductile phase (e.g. Starkey & Cutforth 1978), although in the case of quartz-mica, deformed experimentally in axial compression, part of the reduction in texture in the composite can be attributed to grain boundary sliding on mica, which undergoes grain size reduction (Fig. 18) (Wenk *et al.* 1990). As Handy (1990) has shown, the amount and preferred orientation of 'hard' phases greatly influences the rheology of polymineralic rocks.

Much of the confirmation of a theory relies on comparison of textures in experiments and model simulations. Microstructural observations can directly provide information on the homogeneity of the deformation, and thus the suitability of a plasticity model. For example, it is often observed that in deformed quartzites some grains are strongly deformed and flattened whereas others appear as undeformed porphyroclasts (Fig. 19a). Indeed Tullis *et al.* (1973), Bouchez (1977) and Law (1986) described in quartz anisotropic grain shapes which correlate with the orientation. Takeshita & Wenk (1988) and Wenk *et al.* (1989b) have shown that strongly deformed grains in those reports are in orientations with a small Taylor factor, i.e. they are oriented favorably for slip on soft systems. In such deformed materials which display a large variation in aspect ratios, the strict Taylor theory cannot apply.

Even more direct evidence about heterogeneity comes from observations of dislocations by transmission electron microscopy. While data which correlate crystal orientations and TEM data are scarce it has been documented for polycrystals of experimentally deformed dolomite that in general those slip systems are active which are expected from the crystal orientation; but there are many exceptions. Stresses projected from neighboring grains activate slip systems which are not in accord with the macroscopic stress. There is evidence for tangling of dislocations and even cataclasis along grain boundaries (Fig. 19b). In most crystals only three or fewer slip systems could be identified. Again, these microstructural observations indicate that conditions of homogeneity of strain are not satisfied. We must expect to find deviations from the predictions of the Taylor

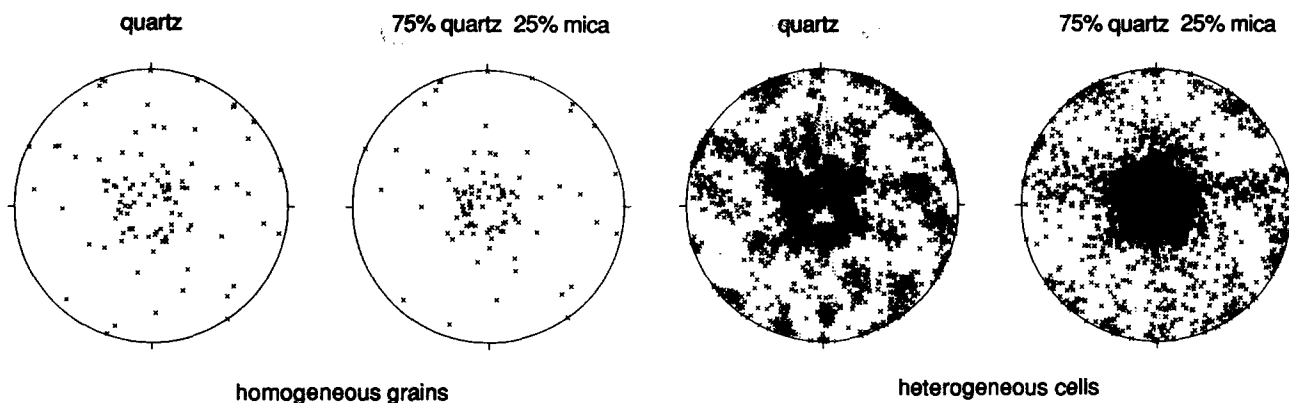


Fig. 16. Deformation of pure quartz and a 75% quartz–25% mica mixture in axial compression to 50% strain according to the VPSC theory. Critical shear stresses are the same as for Fig. 9 (model β). (0001) pole figures with the compression axis in the center. The figure compares results from a uniform grain scheme (intragranular heterogeneity) (left) with grains divided into heterogeneous cells (intergranular heterogeneity) (right). Equal-area projection (Canova *et al.* in press).

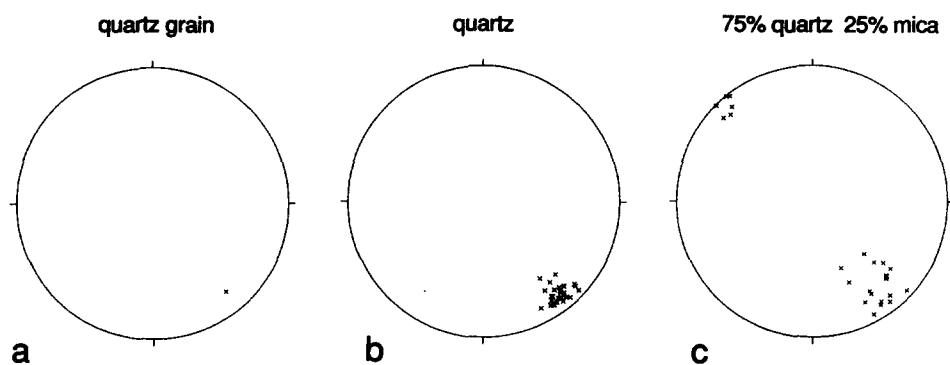


Fig. 17. Comparison of *c*-axis orientation of (a) a single quartz grain in the uniform grain scheme (from Fig. 16) splitting into subgrains in a cell scheme for (b) pure quartzite and (c) when 25% mica is added. Equal-area projection.

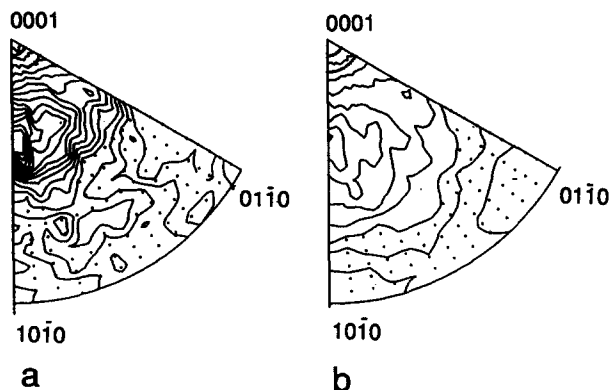


Fig. 18. (a) Pure quartz and (b) 75% quartz–25% mica mixture deformed experimentally by J. Tullis in axial compression to 40% strain at 800°C and a strain rate of 10^{-5} s^{-1} . Inverse pole figures of the compression axis in equal area projection. Minima are dotted. Note the similarity in pattern but the difference in strength of preferred orientation.

theory in mineral systems, even though those predictions appear to be a good first approximation.

OTHER REFINEMENTS OF PLASTICITY MODELS

There are other refinements to the Taylor theory that address real properties of materials. Microstructural

work hardening may modify critical shear stresses as deformation proceeds. Since weak systems on which most of the deformation occurs may harden more quickly, the polycrystal becomes more isotropic with increasing deformation and the Taylor model becomes more applicable. Experimental information on hardening, particularly latent hardening—i.e. the influence of active systems in hardening non-active systems—is unavailable for most minerals. For halite deformed at low temperature, simulations display a strong influence of hardening on texture development (Wenk *et al.* 1989b); in the case of quartz, hardening has a less significant effect (Wenk *et al.* 1989a).

Another important consideration is the strain rate sensitivity with respect to stress. The original Taylor theory assumed a rigid–plastic behavior; i.e. no deformation occurs until the critical resolved shear stress is reached, at which point deformation is instantaneous. It has been mentioned above that such a behavior is unrealistic. A viscoplastic power law is often used to describe the strain rate ($\dot{\gamma}$) sensitivity to the stress τ ($\dot{\gamma} = A\tau^n$). For a stress exponent $n = \infty$ such a law describes rigid–plastic behavior; for $n = 1$ it describes a viscous Newtonian fluid. In between it applies to viscoplastic behavior. Note that with such a description the behavior for stresses above the critical resolved shear stress τ_c is meaningless. For metals with stress exponents between 50 and 100 deviations from rigid–plastic behav-

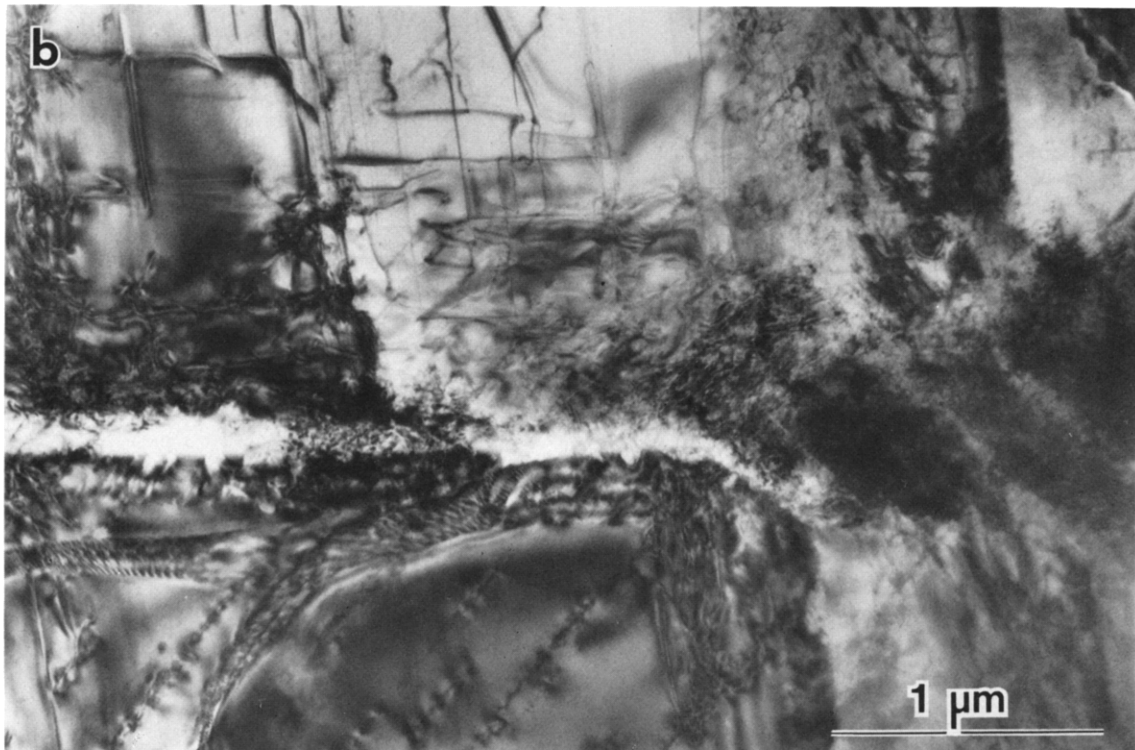


Fig. 19. Microstructures illustrating heterogeneous deformation on a local scale. (a) Photomicrograph of a quartz mylonite from the Bergell Alps with many highly flattened and some almost undeformed grains. Crossed polars. (b) TEM electron micrograph of dislocation structures in experimentally deformed polycrystalline dolomite with dense tangles and some microfracturing along grain boundaries. (Courtesy of D. J. Barber.)

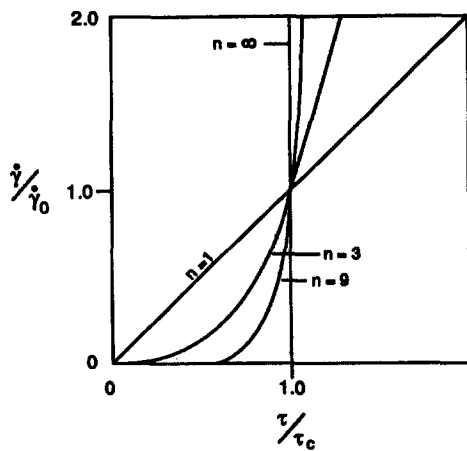


Fig. 20. Strain rate $\dot{\gamma}$ vs shear stress τ plot for rigid plastic ($n = \infty$), Newtonian viscous ($n = 1$) and a power law viscoplastic behavior. Note that in viscoplastic deformation crystals begin to deform below the critical shear stress τ_c but at a lower rate. The behavior above the critical shear stress is physically meaningless.

ior are small. But for many minerals with stress exponents between 3 and 10 the strain rate sensitivity is significant. Crystals deform at low stresses, below the critical resolved shear stress, albeit more slowly (Fig. 20). In viscoplastic deformation all slip systems are active in each crystal at all times, but some only to a very small degree. Again, as with hardening, taking account of strain rate sensitivity causes strain to be distributed more uniformly over many slip systems, leading to a reduction in plastic anisotropy. Strain rate sensitivity is not very significant for texture development in minerals such as calcite, halite and olivine. However, for quartz the influence on texture is profound (Fig. 21; see also Wenk *et al.* 1989a). The viscoplastic texture is smoother than the rigid-plastic texture, corresponding better to natural quartz textures, and, in simple shear, rotations are more uniform so that texture maxima develop more slowly both in Taylor and VPSC simulations (Fig. 9, right side).

In the foregoing discussion texture development has been emphasized. This is one important parameter in polycrystal plasticity and a very sensitive indicator of

strain history. But development of preferred orientation is merely an expression of slip system activity which influences other parameters such as yield stress and changes of yield stress with the deformation history. Whether a material hardens or softens due to the development of preferred orientation often cannot be determined experimentally for complex strain paths, including geologically important pure shear and simple shear deformation. However, it can be easily calculated, and we can determine, for example, that a quartzite subjected to a macroscopic strain at low temperature deforms more easily in axial compression (flattening), and at high temperature in axial tension (elongation) (Fig. 22), which is consistent with observations in naturally deformed metamorphic rocks (e.g. Bouchez 1977). The internal 'plastic' anisotropic structure of a rock may determine along which deformation path a rock may continue to deform. If texture development produces weakening, there is a chance for instabilities (e.g. White *et al.* 1980). Deformation may concentrate in narrow zones or bands such as in mylonite zones, though commonly composition is also an important factor, most mylonitic rocks being quartz-rich and probably hydrolytically weakened. The anisotropy and heterogeneity of macroscopic flow in polycrystalline aggregates is directly related to the microscopic deformation behavior and the detailed analysis of polycrystal plasticity therefore has implications for the interpretation of large-scale deformations of tectonic importance.

EFFECT OF RECRYSTALLIZATION

Closely linked to deformation by slip is recrystallization, and a short discussion is appropriate because recrystallization—whether static or dynamic—is a major process by which preferred orientation develops in rocks (Green *et al.* 1970). The driving force for any recrystallization is internal strain. There is a general tendency for a grain boundary to move towards the more highly deformed grain, thereby reducing the free energy of the system. This results in a preferential selection of grains with no, or few dislocations which dominate the fabric.

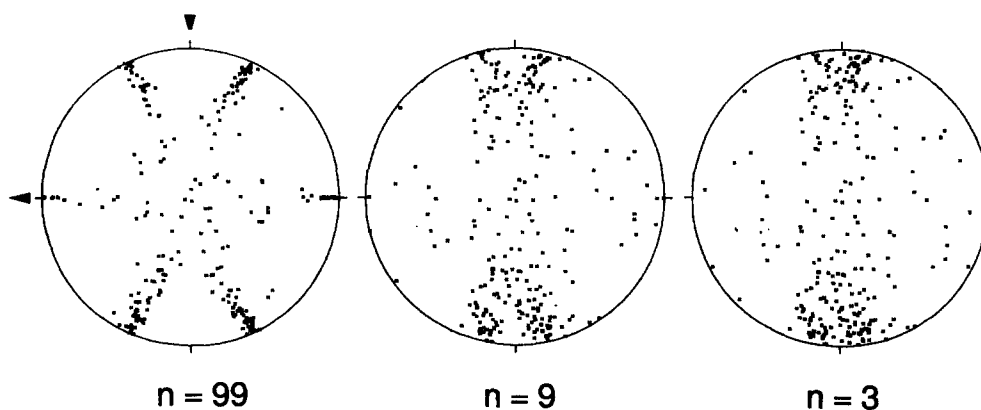


Fig. 21. Quartz *c*-axis pole figures predicted with a viscoplastic Taylor theory for pure shear deformation to 100% equivalent strain. Critical shear stresses correspond to high temperature with prismatic slip dominating (same as Fig. 9). Illustrated is the influence of the stress exponent on texture development, where $n = 3$ is the experimentally determined value for quartzite (e.g. Heard & Carter 1968, Koch *et al.* 1989). Equal-area projection (Wenk *et al.* 1989a).

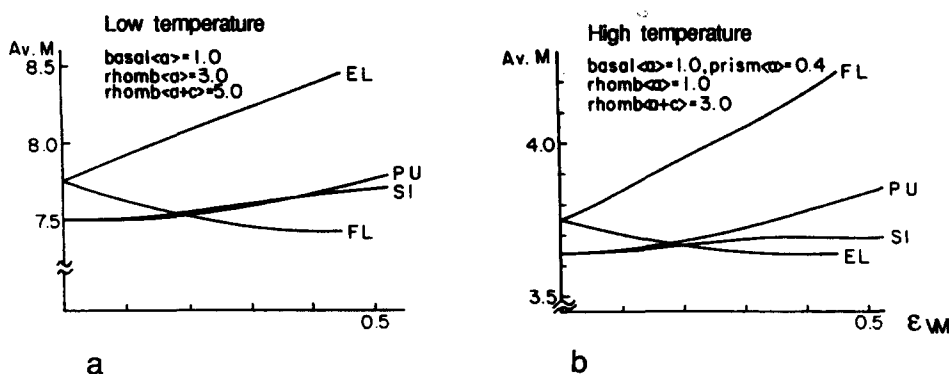


Fig. 22. Changes in the average Taylor factor M with equivalent Von Mises ϵ_{VM} strain for quartzite. M is a measure of the plastic strength of the polycrystal due to crystal orientations and critical resolved shear stresses. It is based on the Taylor theory with five slip systems operating in each grain. Deformation is in elongation EL (axial tension), pure shear PU, simple shear SI, and flattening FL (axial compression) at low temperature (a) where basal slip is easiest and at high temperature, (b) where prismatic slip dominates (Takeshita & Wenk 1988). The material softens at low temperature during flattening, at high temperature during elongation.

This has been impressively documented by *in situ* observations of recrystallization during deformation in two-dimensional model systems such as octachloropropane, camphor and NaNO_3 (e.g. Tungatt & Humphreys 1981, Means 1983, Jessell 1986, Means & Jessell 1986, Urai *et al.* 1986, Means & Ree 1988). In these experiments there is a general tendency for less deformed grains to replace more highly deformed grains which are more favorably oriented for slip. Similar features have been described in experimentally deformed dunite where olivine grains in favorable orientations for slip have higher dislocation densities and recrystallize first (Karato 1987).

Unfortunately recrystallization is much less understood than deformation by slip (e.g. Gottstein & Mecking 1985, Doherty *et al.* 1988). There is no agreement among metallurgists whether preferred nucleation or preferred grain growth governs recrystallization in which materials and under which conditions, even for fcc metals. No general model is available to predict texture development. In all materials investigated so far (including quartz, Hobbs 1968), there appears to be a close orientation relationship between host and newly nucleated grains and therefore a recrystallization texture generally emphasizes some components of the deformation texture.

However, nucleation of new grains involves a *discontinuous* change of orientation that is not provided for by predictions of pure plasticity theories, though the new orientations are probably related to the slip history. Hobbs (1968) demonstrated convincingly that nucleation in deformed quartz crystals, recrystallized during deformation or during annealing, occurs in the most deformed regions (such as deformation bands). The observed orientations of neoblasts are consistent with the nuclei having the orientations of the most highly deformed and rotated regions of the host, such as kinks and subgrains. In annealing, c -axes of neoblasts are inclined up to 40° to that of the host crystal, with those at 20 – 40° to the host growing most rapidly. During syntectonic recrystallization, c -axes of new grains tend to lie at 30 – 50° to that of the host. This is consistent with exten-

sive TEM observations of *in situ* recrystallization in metals that new grains nucleate in regions of highest dislocation density, which rapidly form subgrains and grow at the expense of surrounding strained matrix, sometimes by subgrain coalescence. During dynamic (or syntectonic) recrystallization these new grains, which are initially dislocation-free and therefore relatively soft, deform continuously by slip and climb.

Jessell (1988a,b) and Jessell & Lister (1990) have devised a model for texture development during recrystallization, assuming that grain boundary migration is the dominant mechanism, and applied it to quartz and ice. In this case the less deformed grains consume the more highly deformed grains. Jessell (1988a) assessed the degree of deformation of individual grains with the Taylor factor. A more quantitative measure would be the accumulated plastic work which can be evaluated with the self-consistent theory.

Wenk *et al.* (1989a) have noted that in VPSC simulations of quartz deformed in pure shear the most highly deformed grains (Fig. 23a) have the same orientation as recrystallized grains in quartzite mylonites, i.e. a concentration of c -axes in the intermediate fabric direction (Fig. 23b). They concluded that in such a case nucleation of new grains may occur in the same orientation as those grains which have accumulated the largest plastic work. These grains would grow and dominate the ultimate texture pattern. This is in agreement with the observations of Hobbs (1968).

A similar case may exist for halite deformed in extension. Strongly deformed grains have extension axes near (001) (Fig. 24) (Wenk *et al.* 1989b). The texture of recrystallized extruded halite also has a maximum near (001) (Fig. 24b) (Skrotzki & Welch 1983).

These arguments about recrystallization are still very qualitative but they suggest that refined polycrystal plasticity theories may also be a key to understanding the role of recrystallization in the development of preferred orientation. As with slip, the problem of polyphase materials will be profound in the case of polymineralic rocks because chemical and physical differences will inhibit grain boundary migration.

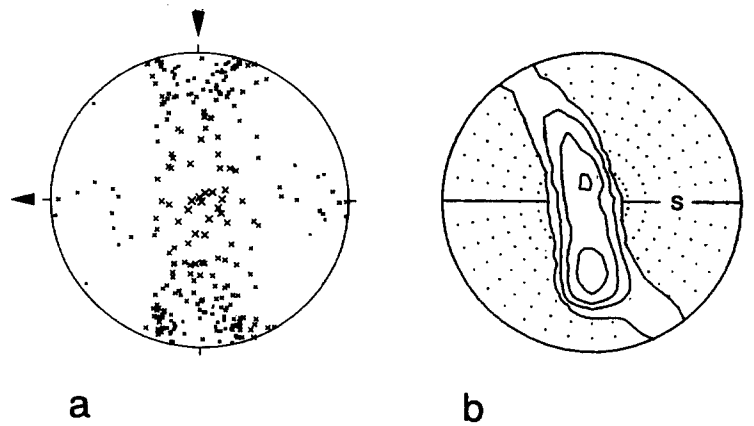


Fig. 23. (0001) pole figures for quartz. (a) VPSC prediction for pure shear deformation to 50% equivalent strain. Symbol size is proportional to the accumulated plastic work. Critical resolved shear stresses are the same as for Fig. 9 (model β of Wenk *et al.* 1989a). (b) Naturally deformed recrystallized quartz mylonite from the northern Bergell Alps (cf. Fig. 11). Schistosity plane (s) and lineation (l) are shown. Equal-area projection, dotted below 1 m.r.d. Note that grains with the highest accumulated plastic work are in the same orientation as those in the recrystallized tectonite.

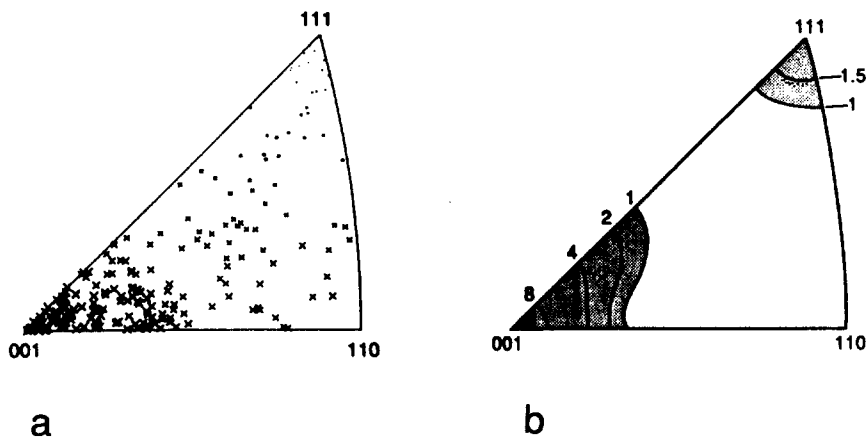


Fig. 24. Inverse pole figures for halite deformed in axial extension. (a) Viscoplastic self-consistent prediction for 100% deformation. Symbol size is proportional to accumulated plastic work. (b) Experimentally extruded recrystallized halite, after Skrotzky & Welch (1983). Maxima are shaded. Stereographic projection (compare with Fig. 13). The orientation of the highly deformed grains at (001) in (a) corresponds well with the recrystallized texture.

DISCUSSION

Mineral preferred orientation patterns have long been used to analyze geological history. In this endeavor the symmetry principle, which states that the symmetry of a texture has to be a subgroup of the symmetry of the starting texture and of the strain path (Paterson & Weiss 1961), has been the most useful guide for interpreting deformation textures and has been widely applied. For example, pole figures with orthorhombic symmetry are taken to indicate coaxial deformation, whereas monoclinic or triclinic pole figures imply a component of non-coaxial deformation. The symmetry principle makes no assumptions about mechanisms and has therefore a very general validity. However with the advent of polycrystal plasticity theories, interpretations can be more quantitative in specific cases. If deformation occurs by slip then the amount of simple shear can be estimated from the degree of asymmetry (e.g. in calcite the angular displacement of the *c*-axis maximum from the pole of the

schistosity, Wenk *et al.* 1987b). The pattern of preferred orientation is a result of the whole strain history, and not just the finite strain. For example, a cube of aluminum which is alternately compressed in the three cube normals develops strong preferred orientation, even though the finite strain ellipsoid never deviates much from a sphere (Takeshita *et al.* 1989). Also limestone which is first compressed and then extended to regain its original shape, is textured (Wenk *et al.* 1986). However with a uniform strain path, preferred orientation increases with increasing strain and strong textures generally indicate large deformations. But to use preferred orientation quantitatively to infer the finite strain is unjustified unless some very restrictive assumptions are made (e.g. Ribe 1989, Ribe & Yu 1991).

Since critical resolved shear stresses on different slip systems may change differently with temperature (in quartz, basal slip is preferred at low temperature and prismatic slip at higher temperature; in calcite, *e*-twinning dominates at low temperature and *r*-slip at

higher temperature), the texture type may yield clues about the metamorphic conditions during tectonism (e.g. Lister & Hobbs 1980). But convincing interpretations are only possible for carefully selected cases and only in conjunction with detailed structural investigations. The Saxony granulites are a good example of oversimplified interpretations of quartz fabric diagrams (e.g. Behr 1968 and Lister & Dornsiepen 1982 interpret a regional texture change as due to temperature, while Hofmann 1974 attributes it to a change in strain regime).

Another complication in the interpretation of geological textures is that the mechanisms of deformation and of texture development may change in the course of the geological history. For example in a mylonite zone rocks may initially deform by dislocation glide, then recrystallize, and with a reduced grain size grain boundary sliding may become the dominant mechanism. Some deformations do not produce preferred orientation. They include diffusion (e.g. during dislocation climb) and cataclasis (except if crystals are non-equiaxed, Tullis & Yund 1987). Lack of preferred orientation in deformed rocks is often a good indication that mechanisms such as superplasticity through grain boundary sliding have operated (e.g. Schmid *et al.* 1977). Also solution transfer ('pressure solution', e.g. Robin 1978) generally does not produce significant preferred orientations although in special cases a mechanism due to anisotropic dissolution and growth rates in a stress field may give rise to preferred orientation (Tullis 1989).

If grains have initially a non-equiaxed shape, then there will be passive rotations which produce preferred orientation. Deformation of rigid particles in a viscous medium has been modelled by Jeffery (1923) and geological applications were recently reviewed by Oertel (1983). These rigid-body rotations are in general taken into account in polycrystal plasticity models.

Since the time of Sander and Schmidt, the theory of polycrystal plasticity and texture development has seen great advances. Texture development during deformation is no longer a realm of intuitive perception but of rigorous analysis. While the physical framework is pretty well established, the mechanical solution is extremely complicated, requiring approximations and leaving space not only for inaccuracies but also for errors. Progress will probably not come from grandiose new ideas, but from painstaking refinements, many of them empirical and requiring comparison with experiments which have not yet been done. For each mineral and for each temperature, pressure and strain-rate regime, it needs to be decided which theory—emphasizing compatibility, equilibrium, or both—is most suitable to model deformation of rocks, and whether the behavior changes with progressive strain. In the future, models will have to be developed which take better account of microstructure and combinations of different mechanisms such as slip, climb, recrystallization and grain boundary sliding, particularly in polymineralic rocks. Heterogeneous deformation on all scales will no doubt become an important topic of future research and besides classical polycrystal plasticity

theory, finite element methods will become increasingly used (e.g. Mathur & Dawson 1989, McHugh *et al.* 1989, Mathur *et al.* 1990).

In conclusion, we would like to emphasize again the importance of polycrystal plasticity theory for the understanding of preferred orientation patterns which developed during deformation. We have tried to dispel some misconceptions and to illustrate some of the dilemmas which need to be addressed in the future, i.e. the balance between stress equilibrium and strain compatibility for modelling deformation and the balance between nucleation and grain growth for modelling recrystallization. In this task close collaboration with materials scientists and physicists will be essential, and creative but unfounded intuition should be discouraged. In spite of difficulties, prospects for texture analysis in the earth sciences are exciting and it will remain an important and challenging subject to pursue through field observations, measurements, experiments, and theoretical analysis.

Acknowledgements—H.-R. Wenk is appreciative to the Deutsche Mineralogische Gesellschaft (through G. Will) and to the American Geophysical Union (through S. Kirby) for invitations to give keynote papers which became the basis of this discussion. But it was only through the enthusiastic encouragement, discussion and critical comments of colleagues—among them K. Bennett, G. Canova, M. W. Jessell, U. F. Kocks, R. Law, L. Ratschbacher, S. Treagus and J. Tullis—that this project has come to completion. Support from NSF EAR 9017237, UC-IGPP at Los Alamos and at Livermore, and INCOR are gratefully acknowledged.

REFERENCES

- Adams, B. 1986. Description of the intercrystalline structure distribution in polycrystalline materials. *Metal. Trans.* **17A**, 2199–2207.
- Asaro, R. J. & Needleman, A. 1985. Texture development and strain hardening in rate dependent polycrystals. *Acta metall.* **33**, 923–953.
- Behr, H.-J. 1968. Zur tektonischen Analyse magmatischer Körper unter besonderer Berücksichtigung des Quarz Korngefüges II. *Freiberger Forsch.* **C219**, 33–98.
- Bishop, J. F. W. & Hill, R. 1951. A theory of plastic distortion of a polycrystalline aggregate under combined stresses. *Phil. Mag.* **42**, 414–427.
- Bouchez, J. L. 1977. Plastic deformation of quartzites at low temperature in an area of natural strain gradient. *Tectonophysics* **39**, 25–50.
- Budiansky, B. & Wu, T. T. 1962. Theoretical prediction of plastic strain of polycrystals. In: *Proc. 4th Congr. Appl. Mech.*, 1175–1185.
- Bunge, H. J. 1965. Zur Darstellung allgemeiner Texturen. *Z. Metall.* **56**, 872–874.
- Bunge, H. J. 1969. *Mathematische Methoden der Texturanalyse*. Akademie, Berlin.
- Calnan, E. A. & Clews, C. J. B. 1950. Deformation textures of face centered cubic metals. *Phil. Mag.* **41**, 1085–1100.
- Canova, G. R., Frassengeas, C., Molinari, A. & Kocks, U. F. 1988. Effect of rate sensitivity on slip systems activity and lattice rotation. *Acta metall.* **39**, 1961–1970.
- Canova, G. R., Kocks, U. F. & Jonas, J. J. 1984. Theory of torsion texture development. *Acta metall.* **32**, 211–226.
- Canova, G. R., Kocks, U. F., Tomé, C. N. & Jonas, J. J. 1985. The yield surface of textured polycrystals. *J. Mech. Phys. Solids* **33**, 371–397.
- Canova, G. R., Wenk, H.-R. & Molinari, A. In press. Simulation of texture development in polyphase materials. *Acta metall.*
- Cotta, B. 1846. *Grundriss der Geognosie und Geologie*. Arnold, Dresden.
- Dell'Angelo, L. N. & Tullis, J. 1986. A comparison of quartz *c*-axis preferred orientations in experimentally deformed aplites and quartzites. *J. Struct. Geol.* **8**, 683–692.
- Dell'Angelo, L. N. & Tullis, J. 1989. Fabric development in experimentally sheared quartzites. *Tectonophysics* **169**, 1–21.
- D'Hallo, O. J. J. 1833. *Introduction à la Géologie*. Levrault, Paris.
- Doherty, R. D., Gottstein, G., Hirsch, J., Hutchinson, W. B., Lücke,

- K., Nes, E. & Wilbrandt, P. 1988. Report on panel recrystallization textures: mechanisms and experiments. In: *Proc. 8th Int. Conf. on Textures of Materials* (edited by Kallend, J. S. & Gottstein, G.). TMS-AIME, Warrendale, Pennsylvania, 563–572.
- Etchecopar, A. 1977. A plane kinematic model of progressive deformation in a polycrystalline aggregate. *Tectonophysics* **39**, 121–139.
- Etchecopar, A. & Vasseur, G. 1987. A 3D kinematic model of fabric development in polycrystalline aggregates: Comparisons with experimental and natural examples. *J. Struct. Geol.* **9**, 705–717.
- Franssen, R. C. M. & Spiers, C. J. 1990. Deformation of polycrystalline salt in compression and in shear at 250°C–350°C. In: *Deformation Mechanisms, Rheology and Tectonics* (edited by Knipe, R. J. & Rutter, E. H.). *Spec. Publs geol. Soc. Lond.* **54**, 201–213.
- Gil Sevillano, J., Van Houtte, P. & Aernoudt, E. 1980. Large strain work hardening and textures. *Prog. Mater. Sci.* **25**, 69–412.
- Gottstein, G. & Mecking, H. 1985. Recrystallization. In: *Preferred Orientation in Deformed Metals and Rocks, An Introduction to Modern Texture Analysis* (edited by Wenk, H.-R.). Academic Press, Orlando, 183–218.
- Green, H. W., Griggs, D. T. & Christie, J. M. 1970. Syntectonic and annealing recrystallization of fine-grained quartz aggregates. In: *Experimental and Natural Rock Deformation* (edited by Paulitsch, P.). Springer, Berlin, 272–335.
- Grewen, J. & Wassermann, G. 1955. Über die idealen Orientierungen einer Walztextur. *Acta metall.* **3**, 354–360.
- Groves, G. W. & Kelley, A. 1963. Independent slip systems in crystals. *Phil. Mag.* **8**, 877–887.
- Handy, M. R. 1990. The solid-state flow of polymineralic rocks. *J. geophys. Res.* **95B**, 8647–8661.
- Heard, H. C. & Carter, N. L. 1968. Experimentally induced “natural” intragranular flow of quartz and quartzite. *Am. J. Sci.* **266**, 1–42.
- Helming, K., Matthies, S. & Vinel, G. W. 1988. ODF representation by means of σ -section. In: *Proc. 8th Int. Conf. Texture of Materials* (edited by Kallend, J. & Gottstein, G.). TMS-AIME, Warrendale, Pennsylvania, 55–60.
- Hobbs, B. E. 1968. Recrystallization of single crystals of quartz. *Tectonophysics* **6**, 353–401.
- Hofmann, J. 1974. Die Quartzteilgefüge von Metamorphiten und Anatexiten, dargestellt am Beispiel des Osterzgebirges (DDR). *Freiberger Forsch.* **C297**.
- Honneff, H. & Mecking, H. 1978. A method for the determination of the active slip systems and orientation changes during single crystal deformation. In: *Proc. 5th Int. Conf. Texture of Materials* (edited by Gottstein, G. & Lücke, K.). Springer, New York, 265–275.
- Hosford, W. F. 1964. Microstructural changes during deformation of [001] fiber-textured metals. *Trans. metall. Soc. AIME* **230**, 12–15.
- Hughes, D. A. & Wenk, H.-R. 1988. The effect of stacking fault energy on the texture of nickel-cobalt solid solutions at large strains. In: *Proc. 8th Int. Conf. Texture of Materials* (edited by Kallend, J. & Gottstein, G.). TMS-AIME, Warrendale, Pennsylvania, 455–460.
- Hutchinson, J. W. 1970. Elastic-plastic behavior of polycrystalline metals and composites. *Proc. R. Soc. Lond.* **A319**, 247–272.
- Hutchinson, J. W. 1976. Bounds and self-consistent estimates for creep of polycrystalline materials. *Proc. R. Soc. Lond.* **A348**, 101–127.
- Jessell, M. W. 1986. Grain boundary migration and fabric development in experimentally deformed octachloropropane. *J. Struct. Geol.* **8**, 527–542.
- Jessell, M. W. 1988a. Simulation of fabric development in recrystallizing aggregates—I. Description of the model. *J. Struct. Geol.* **10**, 771–778.
- Jessell, M. W. 1988b. Simulation of fabric development in recrystallizing aggregates—II. Example model runs. *J. Struct. Geol.* **10**, 779–793.
- Jessell, M. W. & Lister, G. S. 1990. A simulation of the temperature dependence of quartz fabrics. In: *Deformation Mechanisms, Rheology and Tectonics* (edited by Knipe, R. J. & Rutter, E. H.). *Spec. Publs geol. Soc. Lond.* **54**, 353–362.
- Johnsen, A. 1918. Künstliche Schiebungen und Translationen nach Untersuchungen von K. Veit. *Zentbl. Geol.* **19**, 265–266.
- Karato, S. 1987. Seismic anisotropy due to lattice preferred orientation of minerals: Kinematic or dynamic? In: *High-pressure Research in Mineral Physics* (edited by Manghni, M. H. & Syono, Y.). Academic Press, Orlando, 455–471.
- Kern, H. & Wenk, H.-R. 1983. Calcite texture development in experimentally induced ductile shear zones. *Contr. Miner. Petrol.* **83**, 231–236.
- Koch, P. S., Christie, J. M., Ord, A. & George, R. P. 1989. Effect of water on the rheology of experimentally deformed quartzite. *J. geophys. Res.* **94**, 13,975–13,996.
- Kocks, U. F. 1970. The relation between single crystal deformation and polycrystal deformation. *Metall. Trans.* **1**, 1121–1143.
- Kocks, U. F. 1988. A symmetric set of Euler angles and oblique orientation space sections. In: *Proc. 8th Int. Conf. Texture of Materials* (edited by Kallend, J. & Gottstein, G.). TMS-AIME, Warrendale, Pennsylvania, 31–36.
- Kocks, U. F. & Canova, G. R. 1981. How many slip systems and which? In: *Deformation of Polycrystals: Mechanisms and Microstructures, Proc. 2nd RISO Int. Symp.*, Riso National Laboratory, Roskilde, Denmark, 35–44.
- Kröner, E. 1961. Zur plastischen Verformung des Vielkristalls. *Acta metall.* **9**, 155–161.
- Law, R. D. 1986. Relationship between strain and quartz crystallographic fabrics in the Roche Maurice quartzites of Plougastel, Western Brittany. *J. Struct. Geol.* **8**, 493–515.
- Law, R. D. 1990. Crystallographic fabrics: a selective review of their applications to research in structural geology. In: *Deformation Mechanisms, Rheology and Tectonics* (edited by Knipe, R. J. & Rutter, E. H.). *Spec. Publs geol. Soc. Lond.* **54**, 335–352.
- Law, R. D., Schmid, S. M. & Wheeler, J. 1990. Simple shear deformation and quartz crystallographic fabrics: a possible natural example from the Torridon area of NW Scotland. *J. Struct. Geol.* **12**, 29–45.
- Leffers, T. 1981. Microstructures and mechanisms of polycrystal deformation at low temperature. In: *Deformation of Polycrystals: Mechanisms and Microstructures, Proc. 2nd RISO Int. Symp.*, Riso National Laboratory, Roskilde, Denmark, 55–71.
- Lister, G. S. & Dornsiepen, V. F. 1982. Fabric transitions in the Saxony granulite terrain. *J. Struct. Geol.* **4**, 81–92.
- Lister, G. S. & Hobbs, B. E. 1980. The simulation of fabric development during plastic deformation and its application to quartzite: the influence of deformation history. *J. Struct. Geol.* **2**, 355–370.
- Lister, G. S., Paterson, M. S. & Hobbs, B. E. 1978. The simulation of fabric development during plastic deformation and its application to quartzite: The model. *Tectonophysics* **45**, 107–158.
- Lloyd, G. E., Ferguson, C. C. & Law, R. D. 1987. Discriminatory petrofabric analysis of quartz rocks using SEM electron channeling. *Tectonophysics* **135**, 243–249.
- Mainprice, D. & Nicolas, A. 1989. Development of shape and lattice preferred orientation: application to the seismic anisotropy of the lower crust. *J. Struct. Geol.* **11**, 175–189.
- Mainprice, D. & Paterson, M. S. In press. Experimental deformation of flint. *Tectonophysics*.
- Mark, H., Polanyi, M. & Schmid, E. 1923. Vorgänge bei der Dehnung von Zinkkristallen. Allgemeine Beschreibung der Erscheinungen und Untersuchungsmethoden. *Z. Phys.* **12**, 58–116.
- Mathur, K. K. & Dawson, P. R. 1989. On modeling the development of crystallographic textures in bulk forming processes. *Int. J. Plast.* **5**, 67–94.
- Mathur, K. K., Dawson, P. R. & Kocks, U. F. 1990. On modeling anisotropy in deformation processes involving textured polycrystals with distorted grain shape. *Mech. Materials* **10**, 183–202.
- Matthies, S., Wenk, H.-R. & Vinel, G. W. 1988. Some basic concepts of texture analysis and comparison of three methods to calculate orientation distributions from pole figures. *J. appl. Crystall.* **21**, 285–304.
- McHugh, P. E., Varias, A. G., Asaro, R. J. & Shih, C. F. 1989. Computational modeling of microstructures. *Future Generation Comput. Syst.* **5**, 295–318.
- Means, W. D. 1983. Microstructure and micromotion in recrystallization flow of octachloropropane, a first look. *Geol. Rdsch.* **72**, 511–528.
- Means, W. D. & Jessell, M. W. 1986. Accommodation migration of grain boundaries. *Tectonophysics* **127**, 67–86.
- Means, W. D. & Ree, J. R. 1988. Seven types of subgrain boundaries in octachloropropane. *J. Struct. Geol.* **10**, 765–770.
- Molinari, A., Canova, G. R. & Ahzi, S. 1987. A self-consistent approach of the large deformation polycrystal viscoplasticity. *Acta metall.* **35**, 2983–2994.
- Mügge, O. 1883. Beiträge zur Kenntnis der Strukturflächen des Kalkspaths. *Neues Jb. Miner.* **1**, 32–54, 81–85.
- Mügge, O. 1898. Ueber Translationen und verwandte Erscheinungen in Kristallen. *Neues Jb. Miner. Geol. Palaeont.* **1**, 71–162.
- Naumann, C. F. 1850. *Lehrbuch der Geognosie*. Engelmann, Leipzig.
- Oertel, G. 1983. The relationship of strain and preferred orientation of phyllosilicate grains in rocks—review. *Tectonophysics* **100**, 413–447.
- Orowan, E. 1934. Plasticity of crystals. *Z. Phys.* **89**, 605–659.
- Parks, D. M. & Ahzi, S. 1990. Polycrystalline plastic deformation and texture evolution for crystals with less than five independent slip systems. *J. Mech. Phys. Solids.* **38**, 701–724.

- Paterson, M. S. & Weiss, L. E. 1961. Symmetry concepts in the structural analysis of deformed rocks. *Bull. geol. Soc. Am.* **72**, 841–872.
- Polanyi, M. 1925. Deformation von Einkristallen. *Z. Kristallogr.* **61**, 49–57.
- Polanyi, M. 1934. Lattice distortion which originates plastic flow. *Z. Phys.* **89**, 660.
- Ramsauer, H. 1941. Achsenverteilungsanalysen von Quarztektoniten. Unpublished dissertation, University of Innsbruck.
- Reid, C. N. 1973. *Deformation Geometry for Materials Scientists*. Pergamon Press, Oxford.
- Ribe, N. M. 1989. A continuum theory for lattice preferred orientation. *Geophys. J.* **97**, 199–207.
- Ribe, N. M. & Yu, Y. 1991. A theory for the evolution of orientation textures in deformed olivine polycrystals. *J. geophys. Res.* **96B**, 8325–8335.
- Robin, P.-Y. 1978. Pressure solution at grain contacts. *Geochim. cosmochim. Acta* **42**, 1383–1389.
- Roe, R. J. 1965. Description of crystallite orientation in polycrystalline materials. II. General solution to pole figure inversion. *J. appl. Phys.* **36**, 2024–2031.
- Sachs, G. 1928. Zur Ableitung einer Fließbedingung. *Z. Verb. dt. Ing.* **12**, 134–136.
- Sander, B. 1911. Über Zusammenhänge zwischen Teilbewegung und Gefüge in Gesteinen. *Tschermaks miner. petrog. Mitt.* **30**, 281–314.
- Sander, B. 1923. Zur petrographisch-tektonischen Analyse. *Jb. geol. Bundesanst.* **A23**, 215.
- Sander, B. 1930. *Gefügekunde der Gesteine*. Springer, Berlin.
- Sander, B. 1931. Über Beziehungen der Gefügekunde der Gesteine zu G. Becker und zur Materialphysik. *Z. Miner. Geol. Paläont. Abt. A*, 137–146.
- Sander, B. 1950. *Einführung in die Gefügekunde der Geologischen Körper, Vol. 2*. Springer, Vienna.
- Schmid, E. 1928. Zn-normal stress law. In: *Proc. Int. Congr. on Applied Mechanics*, Delft.
- Schmid, S. M., Boland, J. N. & Paterson, M. S. 1977. Superplastic flow in fine grained limestone. *Tectonophysics* **43**, 257–291.
- Schmid, S. M. & Casey, M. 1986. Complete fabric analysis of some commonly observed quartz *c*-axis patterns. In: *Mineral and Rock Deformation: Laboratory Studies—The Paterson Volume* (edited by Hobbs, B. E. & Heard, M. C.). *Am. geophys. Un. Geophys. Monogr.* **36**, 263–286.
- Schmid, S. M., Casey, M. & Starkey, J. 1981. An illustration of the advantages of a complete texture analysis described by the orientation distribution function (ODF) using quartz pole figure data. *Tectonophysics* **78**, 101–117.
- Schmidt, W. 1912. Zum Bewegungsbild liegender Falten. *Verh. geol. Reichsanst.* **3**, 112–119.
- Schmidt, W. 1927a. Untersuchung über die Regelung der Quarzgefüges kristalliner Schiefer. *Fortsch. Miner. Krist. Petrog.* **11**, 27–31.
- Schmidt, W. 1927b. Zur Quarzgefügeregel. *Fortsch. Miner. Krist. Petrog.* **11**, 334–343.
- Schmidt, W. 1931. Kritisches zu Sander's Gefügekunde der Gesteine. *Zentbl. Miner. Geol. Paläont. Abt. A*, 23–27.
- Schmidt, W. 1932. *Tektonik und Verformungslehre*. Borntträger, Berlin.
- Skrotzki, W. & Welch, P. 1983. Development of texture and microstructure in extruded ionic polycrystalline aggregates. *Tectonophysics* **99**, 47–62.
- Spiers, C. J. 1979. Fabric development in calcite polycrystals deformed at 400°C. *Bull. Mineral.* **102**, 282–289.
- Starkey, J. & Cutforth, C. 1978. A demonstration of the interdependence of the degree of quartz preferred orientation and the quartz content of deformed rocks. *Can. J. Earth Sci.* **15**, 841–847.
- Takeshita, T., Kocks, U. F. & Wenk, H.-R. 1989. Strain path dependence of texture development in aluminum. *Acta metall.* **37**, 2595–2611.
- Takeshita, T., Tomé, C. N., Wenk, H.-R. & Kocks, U. F. 1987. Single crystal yield surface for trigonal lattices: Application to texture transitions in calcite polycrystals. *J. geophys. Res.* **B92**, 12,917–12,920.
- Takeshita, T. & Wenk, H.-R. 1988. Plastic anisotropy and geometric hardening in quartzites. *Tectonophysics* **149**, 345–361.
- Takeshita, T., Wenk, H.-R., Molinari, A. & Canova, G. 1990. Simulation of dislocation assisted plastic deformation in olivine polycrystals. In: *Deformation Processes in Minerals, Ceramics and Rocks* (edited by Barber, D. J. & Meredith, P. G.). Allen & Unwin, London, 365–377.
- Taylor, G. I. 1934. The mechanism of plastic deformation of crystals. *Proc. R. Soc. Lond.* **145**, 362–404.
- Taylor, G. I. 1938. Plastic strain in metals. *J. Inst. Metals* **632**, 307–324.
- Taylor, G. I. & Elam, C. F. 1923. The distortion of an aluminium crystal during a tensile test. *Proc. R. Soc. London*, **102**, 643–667.
- Toth, L. S., Gilormini, P. & Jonas, J. J. 1988. Effect of the rate sensitivity of torsion textures. *Acta metall.* **36**, 3077–3091.
- Trommsdorff, V. & Wenk, E. 1963. Diskussion eines Zwillingsgefüges durch Achsenverteilungsanalyse an Quarz eines Tessinergneisses. *Schweiz. miner. petrog. Mitt.* **43**, 687–700.
- Tullis, T. E. 1989. Development of preferred orientation due to anisotropic dissolution/growth rates during solution-transfer creep. *Eos* **70**, 457–458.
- Tullis, J., Christie, J. M. & Griggs, D. T. 1973. Microstructure and preferred orientations of experimentally deformed quartzites. *Bull. geol. Soc. Am.* **84**, 297–314.
- Tullis, J. & Yund, R. A. 1987. Transition from cataclastic flow to dislocation creep of feldspar: mechanisms and microstructures. *Geology* **15**, 606–609.
- Tungatt, P. O. & Humphreys, F. J. 1981. An *in situ* optical investigation of the deformation behavior of sodium nitrate—an analogy for calcite. *Tectonophysics* **78**, 661–675.
- Turner, F. J. 1948. *Mineralogical and Structural Evolution of the Metamorphic Rocks. Mem. geol. Soc. Am.* **30**.
- Turner, F. J., Griggs, D. T., Clark, R. H. & Dixon, R. H. 1956. Deformation of Yule marble, part VII: development of oriented fabrics at 300°C–400°C. *Bull. geol. Soc. Am.* **67**, 1259–1294.
- Turner, F. J., Griggs, D. T. & Heard, H. C. 1954. Experimental deformation of calcite crystals. *Bull. geol. Soc. Am.* **65**, 883–934.
- Urai, J. L., Means, W. D. & Lister, G. S. 1986. Dynamic recrystallization of minerals. *Am. Geophys. Un. Geophys. Monogr.* **38**, 161–199.
- Van Houtte, P. & Wagner, F. 1985. Development of preferred orientation by slip and twinning. In: *Preferred Orientation in Deformed Metals and Rocks: An Introduction to Modern Texture Analysis* (edited by Wenk, H.-R.). Academic Press, Orlando, 233–265.
- Veit, K. 1922. Künstliche Schiebungen und Translationen in Mineralien. *Neues Jb. Miner. Beil. B.* **45**, 121–148.
- Virnich, K. H., Pospiech, J., Flemmer, A. & Lücke, K. 1978. On the analysis of orientation distributions functions by superposition of Gauss type scattering functions. In: *Proc. 5th Int. Conf. Textures of Materials*, Vol. 1 (edited by Gottstein, G. & Lücke, K.). Springer, Berlin, 129–136.
- Wassermann, G. & Grewen, J. 1962. *Texturen Metallischer Werkstoffe*. Springer, Berlin.
- Weissenberg, K. 1925. Zur Systematik und Theorie der Wachstums- und Deformationsstrukturen. *Z. Kristallogr.* **61**, 58–74.
- Wenk, H.-R. 1965. Gefügestudie an Quarzknauern und linsen der Tessiner Kulmination. *Schweiz. mineral. petrog. Mitt.* **45**, 468–516.
- Wenk, H.-R., Bennett, K., Canova, G. & Molinari, A. 1991. Modelling plastic deformation of peridotite with the self-consistent theory. *J. geophys. Res.* **96B**, 8337–8349.
- Wenk, H.-R., Bunge, H. J., Kallend, J. S., Lücke, K., Matthies, S., Pospiech, J. & Van Houtte, P. 1987a. Orientation distributions. Representation and determination. In: *Proc. 8th Int. Conf. Texture of Materials* (edited by Kallend, J. & Gottstein, G.). TMS-AIME, Warrendale, Pennsylvania, 17–30.
- Wenk, H.-R., Canova, G., Molinari, A. & Kocks, U. F. 1989a. Viscoplastic modeling of texture development in quartzite. *J. geophys. Res.* **94B**, 17,895–17,906.
- Wenk, H.-R., Canova, G., Molinari, A. & Mecking, H. 1989b. Texture development in halite: Comparison of Taylor model and self-consistent theory. *Acta metall.* **37**, 2017–2029.
- Wenk, H.-R., Kern, H., Van Houtte, P. & Wagner, F. 1986. Heterogeneous strain in axial deformation of limestone, textural evidence. In: *Mineral and Rock Deformation: Laboratory Studies—The Paterson Volume* (edited by Hobbs, B. E. & Heard, M. C.). *Am. Geophys. Un. Geophys. Monogr.* **36**, 287–295.
- Wenk, H.-R. & Kocks, U. F. 1987. The representation of orientation distributions. *Metall. Trans.* **18A**, 1083–1092.
- Wenk, H.-R., Takeshita, T., Bechler, E., Erskine, B. & Matthies, S. 1987b. Pure shear and simple shear calcite textures. Comparison of experimental, theoretical and natural data. *J. Struct. Geol.* **9**, 731–745.
- Wenk, H.-R., Tullis, J., Heidelbach, F., Canova, G. & Larson, A. 1990. Deformation and preferred orientation of quartz–muscovite mixtures: Experiments and modelling. *Eos* **71**, 1657.
- Wenk, H.-R., Venkatasubramanian, C. S., Baker, D. W. & Turner, F. J. 1973. Preferred orientation in experimentally deformed limestone. *Contr. Miner. Petrol.* **38**, 81–114.
- White, S. H., Burrow, S. E., Carreras, J. & Shaw, N. D. 1980. On mylonites in ductile shear zones. *J. Struct. Geol.* **2**, 175–187.